New-Keynesian DSGE model, durable goods and collateral constraint in a small open economy

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Abstract

This paper studies the role of collateral constraint in determining the direction of movement of durable and nondurable goods characterised by varying degree of price stickiness in a small open economy New-Keynesian DSGE model. Using South African economy as a laboratory I show that a small open economy New-Keynesian DSGE model with collateral constraint is able to reproduce the co-movement of durable and nondurable goods in response to monetary policy shocks observed in data. The stickiness of durable goods' prices is an adequate condition for positive co-movement in durable and nondurable goods sectors. This is due to the fact that entrepreneurs face constant returns to scale and there are no barriers to moving factors of production across sectors.

1 Introduction

This paper studies the role of collateral constraint in determining the direction of movement of durable and nondurable goods characterised by varying degree of price stickiness in a small open economy simulated to South African data over quarterly time frequency. The degree of price stickiness plays a critical role in influencing the intertemporal substitution of durable and nondurable goods' consumption. Empirical evidence, (Bernanke and Gertler,1995; Barsky et. al., 2003; Erceg and Levin, 2006; and Monacelli, 2009) show that the aggregate consumption in both durable and nondurable goods' sectors move in the same direction. Further, response of durable

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goods consumption to all shocks is reported to be greater than that of nondurable goods. However, Barsky et al. (2007) show that in a two-sector economy with flexibly priced durable and sticky priced nondurable goods, the flexibility of prices of durable goods governs the response of aggregate consumption to monetary policy tightening. The insight of this incongruity is the unique element of durable goods in perfect financial markets. Namely, the shadow value of durable goods is approximately constant. The explanation of this constancy can be traced to typically high stock-to-flow ratio of durable goods. Therefore, additional consumption of durable goods, due to the responsiveness to its own user cost, does not improve the total utility. The response is as follows: Whenever durable goods prices are flexible and nondurable goods' price are sticky, monetary policy tightening decreases the relative price of durable goods and nears consistently the user cost of consumption to rise in the durable goods sector and fall in the nondurable goods sector (Monacelli, 2009).

This paper is related to literature on price stickiness and the movement of output in durable and nondurable goods sectors. Erceg and Levin (2006) investigate a sticky-price model with durable and nondurable goods, albeit without a borrowing constraint, to provide evidence that output in the two sectors fall in the face of monetary policy tightening. They also argue that the durable goods sector is more sensitive to interest rate than that of nondurable goods. Barsky et al. (2003, 2007) investigate the pass-through channel of monetary policy tightening. They suggest that the extent of price stickiness in the durable goods sector is crucial in determining direction of movement between durable and nondurable goods. Furthermore, first, sticky durable goods prices is an adequate condition for positive co-movement of the durable and nondurable sectors. Second, existence of long lived durable goods may cause the production of nondurable goods to be unresponsive to monetary policy tightening[1].

This paper is also related to the literature on collateralized borrowing. One of the business cycle facts in collateralized borrowing is negative movement of durable and nondurable goods' outputs in perfect financial markets (Monacelli, 2009). Particularly, each time consumption increases in one sector, it decreases in the other. The insight for this theoretical inconsistency in perfect financial markets is the key feature of durable goods discussed above. The co-movement problem, that is, negative correlation between user cost and relative price of durable goods, is corrected under imperfect financial markets. Monacelli (2009) suggest that, first, existence of collateral constraint on borrowing generates a collateral constraint effect, which

[1] This is because of the stickiness or flexibility in the two sectors and the notion that in the face of countercyclical monetary policy, relative price of durables increases resulting in accumulation of durables – substitution effect from non-durables consumption to durable purchases.

is an increase in shadow value of durables that first, splits the quasi-constancy of durables' shadow value. Second, it raises the user cost of durables that in turn causes a substitution to nondurable consumption. As a result, collateralized borrowing help rectify the inconsistency caused by agency problem in credit markets through the net worth of households and entrepreneurs. Higher net worth implies that borrowers have higher collateral value hence superior ability to borrow while lower net worth means that borrowers' ability to borrow is limited. Accordingly, economic agents' balance sheet provides the basis for movements in nondurables spending and holding of durables. Sterk (2010) re-examine Monacelli (2009) model and reports that presence of collateral constraint does not resolve the co-movement problem, rather, it exuberate the puzzle.

This paper is also related to literature on small open economies. Indeed, Faia and Iliopulos (2011) examine the interaction of monetary policy in a small open economy in which lending is linked to collateral value, and fluctuations in durable prices. They report, among others, that under high financial liberalization, changes in the exchange rate generate fluctuations in collateral value as a result influencing availability of foreign credit. In contrast, Engel and Wang (2011) study a model in which imports and exports movements are pro-cyclical and more volatile to GDP to suggest that durable goods traded internationally explain robust correlation with GDP. Other studies such as Christiansen et al. (2007) measure the purpose of credit frictions on the business cycle and argue that collateralized debt plays a major part in causing positive reaction of consumption to an increase in durable goods' prices. Iacoviello and Minetti (2006) incorporate housing as constraint in the borrowing constraint of a business cycle model to explain positive movement in output levels across countries suggesting that foreign and domestic lenders differ in their ability to enforce borrowing contracts. Nevertheless, none of these studies investigate co-movement of durable and nondurable goods under price stickiness. This study incorporates collateral constraint into a small open economy with varying degree of price stickiness in durable and nondurable goods sectors. The analysis is conducted under two scenarios, a benchmark with perfect financial markets and an alternative with credit market imperfections.

The contribution of this paper is two-fold. First, by studying the co-movement of durable and nondurable goods in a small open economy using South African data. The model economy is characterized by country-specific risk premium that may alter business environment such as changes in asset prices in a specific country. Second, by embedding domestic and foreign borrowing constraints into entrepreneurs' decision making problem.

The main findings of the paper are as follows. Collateral constraint escalates and spreads shocks to the domestic economy. Collateral constraint also plays a crucial role in determining the impact of shocks on durable purchases and nondurable consumption. Responses of durable and nondurable consumption to a contractionary monetary policy shock in a small open economy set up does not reveal the co-movement problem reported in closed economy literature. Therefore, in the face of shocks, flexibility of prices of durables and nondurables do not govern the response of aggregate output in the two sectors.

The rest of the paper is organized as follows. Section 2 presents empirical evidence from an estimated quarterly VAR of South African economy. Section 3 constructs a New Keynesian DSGE small open economy model with collateral constraint. Calibration of the model is outlined in section 4. Section 5 presents the simulated results and discusses the role of collateral constraint in financial markets. A recap of the discussions in presented in section 6.

2 Monetary policy shocks, durable and non-durable consumption: evidence from South Africa

This section presents two stylized attributes of movement in durable and nondurable goods consumption in response to monetary policy shock. That is, aggregate consumption in both durable and nondurable goods sectors move in the same direction and response of durables is greater than that of nondurables. The evidence matches Monacelli (2009) and references therein. I evaluate the effect of monetary policy shock by estimating quarterly vector autoregression (VAR) model for the South African economy. The VAR model consists of seven variables set up as follows; GDP at market prices, domestic durable and nondurable final consumption expenditure, domestic household debt - total mortgage advances plus debt to nonprofit institutions serving households, terms of trade index for foreign trade including gold, percentage average nominal effective exchange rate for 20 trading partners and nominal short-term interest rate - 91 days treasury bills rate.

The VAR system features a constant, a time trend, four lags and monetary policy shock and is estimated over the period 1960Q01-2014Q02[2] using sign restricted VAR model following Uhlig (2005)[3] agnostic procedure that allows data to decide the direction of variables. The agnostic procedure assumes that contractionary monetary

[2] Source of data is South African Reserve Bank: Historical macroeconomic time series information, https://www.resbank.co.za/Research/Statistics/Pages/OnlineDownloadFacility.aspx.

[3] For Uhlig (2005) replication files see: Ambrogio Cesa-Bianchi, (2014). VAR toolbox and examples, https://sites.google.com/site/ambropo/examples.

policy shock does not result in a decrease in the interest rate.

Fig. 1 illustrates estimated impulse response functions on GDP at market prices, domestic durable and nondurable consumption, domestic household debt, terms of trade, nominal effective exchange rate and nominal short-term interest rate. Dotted lines signify one standard error bands. It is evident that whereas GDP at market prices, durable and nondurable consumption, terms of trade and nominal effective interest rate response is a decline in the face of monetary policy shock, domestic household debt rise before falling steadily. These outcomes are robust to different restrictions on constants, lower or larger lags and draws.

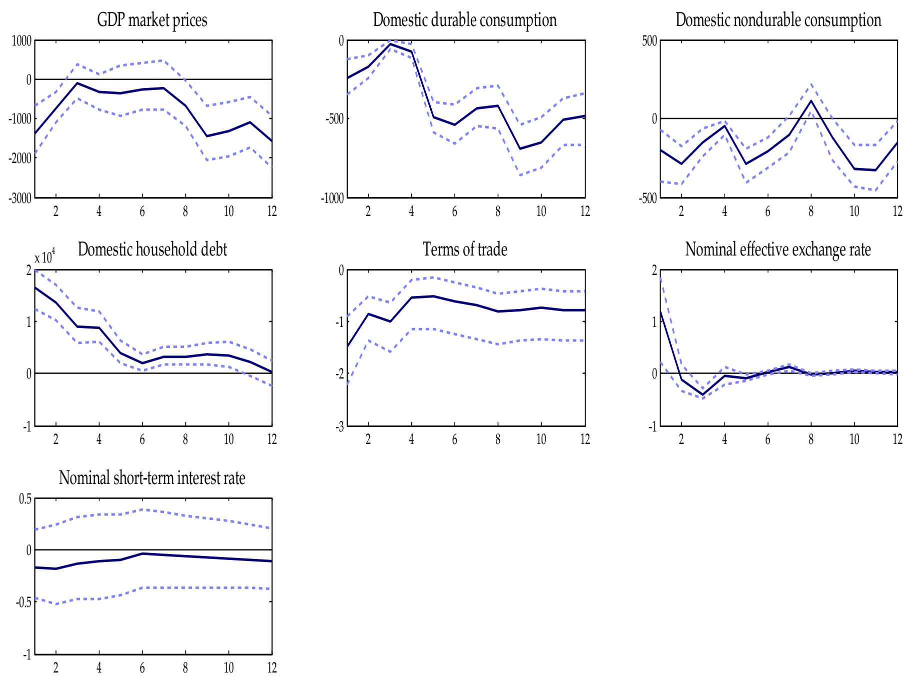

Figure 1: Estimated VAR impulse response functions to monetary policy shock.

3 The model economy

The fundamental framework of this model is a New Keynesian DSGE model in the lines of Monacelli (2009). Nevertheless, the model presented below differs from Monacelli (2009) in the following aspects. First, it studies the co-movement problem in a small open economy using South African data. Given the guiding principles of a small open economy in Gali and Monacelli (2005), Monacelli (2009) study is a closed economy set up. Second, Entrepreneurs face both domestic and foreign collateral constraint. Monacelli (2009) focus only on domestic collateral constraint. The world economy is viewed as a continuum of small open economies represented by a unit interval (0,1). Each country is of measure zero with a country-specific risk premium. Foreign (domestic) policy decision does (do not) affect domestic (foreign) economy conditions. Households supply labour hours of work to entrepreneurs from which they receive wage compensation. Entrepreneurs are monopolistically competitive and hire labour to produce differentiated intermediate goods. Nondurable goods are traded internationally. Prices are sticky in the form of Calvo type price setting.

Borrowing constraint is introduced as follows. Households can lend to domestic and foreign credit markets by purchasing noncontingent bonds. Conversely, entrepreneurs can borrow from domestic and foreign markets by issuing noncontingent bonds.

3.1 Households

There are representative homogenous households that maximise expected lifetime utility function:

$$E_o \sum_{t=0}^{\infty} \beta^t \left[\log C_{l,t} - \frac{\nu N_t^{1+\varphi}}{1+\varphi} \right], \tag{1}$$

where β^t is the households' discount factor, N_t is labour hours, ν is households' preference parameter for hours worked, φ is inverse (unitary) elasticity of labour supply and $C_{l,t}$ is total aggregate consumption,

$$C_{l,t} = \left[(1-\omega)^{\frac{1}{\varkappa}} C_t^{\frac{\varkappa-1}{\varkappa}} + \omega^{\frac{1}{\varkappa}} \widetilde{D}_t^{\frac{\varkappa-1}{\varkappa}} \right]^{\frac{\varkappa}{\varkappa-1}}, \tag{2}$$

where ω is share of durable goods, $\varkappa$ is the elasticity of substitution between durable and nondurable consumption irrespective of country of production. C_t is consumption of nondurable goods and $\widetilde{D}_t$ is stock of durable goods.

Nondurable consumption is specified by a Dixit-Stiglitz aggregator,

$$C_t \equiv \left[(1-\alpha)^{\frac{1}{\eta}} C_{h,t}^{\frac{\eta-1}{\eta}} + \alpha^{\frac{1}{\eta}} C_{f,t}^{\frac{\eta-1}{\eta}} \right]^{\frac{\eta}{\eta-1}}, \tag{3}$$

where $\alpha\epsilon\,[0,1]$ is inversely connected to the degree of home bias, that is, the portion of domestic consumption assigned to imported goods and taken as an index of openness[4] and $\eta > 1$ is a measure of substitutability between domestic and foreign goods.

Durable consumption is specified by,

$$\widetilde{D}_t \equiv \left(D_{t-1} - \frac{\xi}{2} \frac{\left(X_t - \delta D_{t-1}\right)^2}{D_{t-1}} \right), \tag{4}$$

where D_{t-1} is real value of durable stock held in positive amounts. $\frac{\xi}{2}\frac{(X_t-\delta D_{t-1})^2}{D_{t-1}}$ is an adjustment cost. The adjustment cost imposed on durable purchases is meant to control the availability of durables whose prices are not set in advance. This reduces volatility in relative price of durables (Carlstrom and Fuerst, 2006). $X_t = D_t - (1-\delta)\,D_{t-1}$ is investment in durables, ξ is a function that help reduce the volatility of durable purchases in response to shocks[5] and δ is durable goods' depreciation rate.[6]

$C_{h,t}$ and $C_{f,t}$ are given by the CES functions:

$$C_{h,t} = \left(\int_0^1 C_t\,(j)^{\frac{\varepsilon-1}{\varepsilon}}\, dj \right)^{\frac{\varepsilon}{\varepsilon-1}} \text{ and } C_{f,t} = \left(\int_0^1 C_{f,t}\,(j)^{\frac{\gamma-1}{\gamma}}\, dj \right)^{\frac{\gamma}{\gamma-1}},$$

where $j\epsilon\,[0,1]$ denotes a continuum of differentiated final durable and nondurable goods produced by each country. $\varepsilon > 1$ is the elasticity of substitution between variety of goods produced within any given country. γ is substitutability of goods produced in different foreign countries.

The optimal allocation of expenditure between domestic and imported goods after derivation of their respective demand functions, as in Gali and Monacelli (2005), is therefore:

$C_{h,t} = (1-\alpha)\left(\frac{P_{h,t}}{P_t}\right)^{-\eta} C_t$ and $C_{f,t} = \alpha \left(\frac{P_{f,t}}{P_t}\right)^{-\eta} C_t$,

where $P_t \equiv \left[(1-\alpha)\,P_{h,t}^{1-\eta} + \alpha P_{f,t}^{1-\eta}\right]^{\frac{1}{1-\eta}}$ is the consumer price index (CPI).

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[4] See Gali and Monacelli (2005).

[5] See Engel and Wang (2011) and Faia and Iliopulos (2011) for a discussion.

[6] Substituting for Eqs. 2, 3 and 4 into Eq. 1 can be summarised as,

$$E_o \sum_{t=0}^{\infty} \beta_\Gamma^t \left[\log \left[\begin{array}{c} (1-\omega)^{\frac{1}{\varkappa}} \left\{ \left[(1-\alpha)^{\frac{1}{\eta}} C_{h,t}^{\frac{\eta-1}{\eta}} + \alpha^{\frac{1}{\eta}} C_{f,t}^{\frac{\eta-1}{\eta}} \right]^{\frac{\eta}{\eta-1}} \right\}^{\frac{\varkappa-1}{\varkappa}} \\ +\omega^{\frac{1}{\varkappa}} \left\{ \left(D_{t-1} - \frac{\xi}{2} \frac{(X_t - \delta D_{t-1})^2}{D_{t-1}} \right) \right\}^{\frac{\varkappa-1}{\varkappa}} \end{array} \right]^{\frac{\varkappa}{\varkappa-1}} - \frac{\nu (N_t)^{1+\varphi}}{1+\varphi} \right]$$

Households invest in domestic government bonds earning a gross nominal interest rate in period t and own entrepreneurs producing intermediary goods. Households' budget constraint is specified as:

$$C_t+q_tX_t+B_t^h+e_tB_t^f = R_{t-1}^hB_{t-1}^h+\Xi\left(FA_{t-1},\varepsilon_{t-1}^{risk}\right)e_tR_{t-1}^f\frac{e_t}{e_{t-1}}B_{t-1}^f+W_tN_t+T_t, \quad (5)$$

where $q_t \equiv \frac{P_{d,t}}{P_{c,t}}$ is relative price of durable goods. $B_t^h \equiv \frac{b_{t-1}^h}{\pi_{ct}}$ is domestic bonds, and B_t^f is international bonds. e_t is nominal exchange rate. R_{t-1}^f is foreign nominal lending rate on loan contracts. W_t is nominal wage. T_t is lump-sum government transfers/taxes and $\Xi\left(FA_t,\varepsilon_t^{risk}\right)$ is country-specific risk premium function as in Gelain and Kulikov (2009):

$$\Xi\left(FA_{t-1},\varepsilon_{t-1}^{risk}\right) = \left(-\phi_{fa}FA_t+\varepsilon_t^{risk}\right), \quad (6)$$

where FA_t is net foreign asset position of the small open economy. That is, $FA_t \equiv \frac{e_tb_t^f}{P_t^h}$ and $\varepsilon_t^{risk} = \rho_{risk}\varepsilon_{t-1}^{risk}+u_t^{risk}$, $u_t^{risk} \sim i.i.d.\left(0,\sigma_\epsilon^2\right)$. ϕ is country-specific risk premium parameter.

The representative households' first-order conditions for labour supply, holding of domestic and foreign bonds in units of nondurable goods and intertemporal purchase of durable goods are the following:

$$W_t = \frac{\nu}{\left[(1-\alpha)^{\frac{1}{\eta}}+\alpha^{\frac{1}{\eta}}\right]\left[(1-\omega)^{\frac{1}{1-\varkappa}}\right]}C_tN_t^\varphi, \quad (7)$$

$$\frac{1}{C_t} = \beta^tE_t\left(\frac{R_t^h}{C_{t+1}}\right), \quad (8)$$

$$\frac{q_t}{C_t} = \frac{j}{D_t}+\beta^tE_t\left(\frac{q_{t+1}}{C_{t+1}}\right). \quad (9)$$

Eq. 7 is the real wage equation. Eq. 8 outlines the Euler equation for holding of foreign bonds. Eq. 9 gives households' demand for durable goods.

3.2 Entrepreneurs

The supply side of the model is characterized by monopolistic competitive entrepreneurs with Calvo-type price stickiness. Entrepreneurs in each sector hire labour to produce differentiated intermediate goods Y_t financed by borrowing from domestic and foreign households. Entrepreneurs' credit limits is established by the expected price of durable stock used as collateral. The entrepreneurs face a Cobb-Douglas production technology function given by:

$$Y_t = A_t D_{t-1}^{\varpi} N_t^{1-\varpi}, \tag{10}$$

where ϖ is output elasticity of durables. By definition, this is also the share of durables ω (Minetti and Peng, 2013). A_t is a technology shock. That is, $a_t \equiv \log A_t$ and follows an AR(1) process, $a_t = \rho_a a_{t-1} + \epsilon_{a,t}$ where $\epsilon_{a,t} \sim i.i.d.\left(0, \sigma_a^2\right)$.

The entrepreneurs maximize,

$$E_o \sum_{t=0}^{\infty} \zeta^t \log C_t, \tag{11}$$

where $\zeta^t \prec \beta^t$ is the entrepreneurs' discount factor with flow of funds given by,

$$A_t D_{t-1}^{\omega} N_t^{1-\omega} + B_t^h + e_t B_t^f = C_t + q_t X_t + R_{t-1}^h B_{t-1}^h + \Xi\left(FA_{t-1}, \varepsilon_{t-1}^{risk}\right) e_t R_{t-1}^f \frac{e_t}{e_{t-1}} B_{t-1}^f + W_t N_t + T_t. \tag{12}$$

The entrepreneurs also face, first, domestic and foreign borrowing constraints, following Iacoviello and Minetti (2006) and Minetti and Peng (2013) as follows:

$$R_t^h B_t^h \leq E_t\left(\chi_h \sigma q_{t+1} D_t\right), \tag{13}$$

$$R_t^f B_t^f \leq E_t\left[q_{t+1}\left(1-\sigma\right) D_t \left(1 - \frac{1-\chi_f}{qd}\left[q_{t+1}\left(1-\sigma\right) D_t\right]\right)\right], \tag{14}$$

where σ is the share of durable goods used by households as domestic collateral and $1-\sigma$ is the share of foreign collateral. χ is the fraction of durable goods' value that cannot be used as collateral. When collateral constraint is nonbinding, $\chi = 0$. This implies that lenders delay consumption and amass durable stock so that they are fully self-funded hence affecting their steady state. This result in an intratemporal fluctuation in the value of consumption over two periods[7].

[7]Eqs. 13 and 14 can be viewed in terms of limited enforcement of loan contract. Although debt repudiation is in principal possible for the entrepreneur, this option would entail loosing entire current value of the durable stock. Collateral acts as a disincentive against that temptation (Monacelli, 2009). In the neighbourhood of the deterministic steady state, Eqs. 13 and 14 are always satisfied with equality. This assumption allows us to employ local approximation methods when analysis equilibrium dynamics. It therefore requires a bound on the amplitude of the stochastic driving forces in the model. While the collateral constraints are assumed to hold with equality, changes in its tightness are measurable in terms of its resultant shadow value.

Second, a forward looking New Keynesian Phillips curve[8] and an adjustment cost respectively as follows:

$$\pi_{j,t}^{h} = \zeta^{t} E_{t}\left(\pi_{j,t+1}^{h}\right) + \frac{\varepsilon_{j}-1}{\upsilon_{j}}\widehat{mc}_{j,t}, \tag{15}$$

$$\frac{\upsilon_{j}}{2}\left(\frac{P_{t}\left(i\right)}{P_{t-1}\left(i\right)}-1\right)^{2} Y_{t}\left(i\right),$$

where $\widehat{mc}_{j,t}$ is the real marginal cost, and stickiness parameter $\upsilon_{j} \equiv \frac{\theta_{j}(\epsilon_{j}-1)}{(1-\theta_{j})(1-\zeta\theta_{j})} \succeq 0$ is the degree of sectoral nominal price rigidity and determines the size of price adjustment cost. That is, when $\upsilon_{j}=0$, the quadratic cost collapses to a case of fully flexible-prices and is equal to the inverse of steady state markup $\mu \equiv \left(\varepsilon_{j}-1\right)/\varepsilon_{j}$. $\frac{(1-\theta_{j})(1-\zeta\theta_{j})}{\theta_{j}}$ is the slope of the Calvo Phillips curve or adjustment process and θ_{j} is Calvo probability.

The entrepreneurs' first-order conditions for labour demand, durable goods holding, and domestic and foreign borrowing are:

$$W_{t} = \left(1-\varpi\right)\frac{Y_{t}}{N_{t}}, \tag{16}$$

$$\frac{1}{C_{t}} = E_{t}\left(\frac{\zeta^{t} R_{t}^{h}}{C_{t+1}}\right) + \lambda_{t}^{h} R_{t}^{h}, \tag{17}$$

$$\frac{1}{C_{t}} = E_{t}\left(\frac{\zeta^{t} R_{t}^{f}}{C_{t+1}}\right) + \lambda_{t}^{f} R_{t}^{f}\psi_{t}, \tag{18}$$

where λ_{t}^{h} and λ_{t}^{f} are respectively, shadow values of the domestic and foreign borrowing constraints.

The optimal selection of σ equates the marginal benefit of domestic and foreign collateral as follows,

$$\lambda_{t}^{h}\chi_{h} = \lambda_{t}^{f}\widetilde{\chi}_{t+1}^{f}, \tag{19}$$

where $\widetilde{\chi}_{t+1}^{f} \equiv \left[1-2\left(1-\chi_{f}\right)\left(1-\sigma\right)\frac{q_{t+1}D_{t}}{qd}\right]$ is the additional credit allowed when collateral is issued to foreign lenders.

The entrepreneurs' necessary demand for durable goods is,

[8] See Gali and Gertler (1999) for a discussion; Appendix B of Gali and Monacelli (2005) and Supplement 2 of Vasicek and Musil (2006) for illustrated derivation.

$$\frac{q_t}{C_t} = E_t \frac{\zeta^t}{C_{t+1}} \left(\varpi \frac{Y_{t+1}}{D_t} + q_{t+1} \right) + E_t \left[\lambda_t^h \chi_h \sigma q_{t+1} + \lambda_t^f (1 - \sigma) q_{t+1} \widetilde{\chi}_{t+1}^f \right]. \tag{20}$$

Eq. 16 is the real wage equation. Eqs. 17 and 18 outlines the Euler equations for holding of domestic and foreign bonds respectively. Eq. 20 gives entrepreneurs' demand for durable goods. That is, the intertemporal purchase of durable goods which implies that, by purchasing more durables, entrepreneurs shifts consumption from nondurables. This in turn increases current entrepreneurs' credit limits and future consumption of nondurables. In particular, $\lambda_t^h \chi_h \sigma q_{t+1} + \lambda_t^f (1 - \sigma) q_{t+1} \widetilde{\chi}_{t+1}^f$, in Eq. 20 is the shadow value of durable goods representing the marginal utility of relaxing collateral constraint.

Entrepreneurs equate their marginal utility of nondurable consumption to shadow value of durable stock. This depends on direct utility gain on additional units of durables. The expected utility of expanding future consumption, by measure of resale value of the durables in the previous period, and marginal utility of relaxing the collateral constraint proportional to λ_t.

Eq. 20 is a requirement to equating marginal rate of substitution between durables and nondurables. Letting $\frac{d_t}{c_t} = Z_t$ be the user cost of durables, Eq. 20 is expressed in terms of Z_t as follows:

$$Z_t \equiv \frac{1}{q_t} \left\{ E_t \frac{\zeta^t}{C_{t+1}} (\varpi Y_{t+1} + D_t q_{t+1}) + D_t E_t \left[\lambda_t^h \chi_h \sigma q_{t+1} + \lambda_t^f (1 - \sigma) q_{t+1} \widetilde{\chi}_{t+1}^f \right] \right\}. \tag{21}$$

3.3 Terms of trade

Bilateral terms of trade between the domestic and foreign economy is defined as follows. Let terms of trade $S_{i,t} = \frac{P_t^f}{P_t^h}$. This is the price of nondurable goods produced in foreign country i in terms of home country. The effective terms of trade is given by:

$$S_t \equiv \frac{P_{f,t}}{P_{h,t}} = \left(\int_0^1 S_{i,t}^{1-\gamma} di \right)^{\frac{1}{1-\gamma}}. \tag{22}$$

Dividing through households demand functions yields the relation, $\frac{C_{f,t}}{C_{h,t}} = \frac{\alpha}{(1-\alpha)} S_t^{\eta}$. The CPI inflation for nondurables around a steady state that satisfies a purchasing power parity condition is:

$$\pi_t = \pi_{h,t} \frac{\left(\alpha + (1-\alpha) S_t^{1-\eta}\right)^{\frac{1}{1-\eta}}}{\left(\alpha + (1-\alpha) S_{t-1}^{1-\eta}\right)^{\frac{1}{1-\eta}}}. \tag{23}$$

3.4 Real exchange rate

Exchange rate dynamics is derived from Eq. 22 as follows. The index of openness and terms of trade are the gap between the two measures of change in price as a fraction of percentage change in terms of trade. Law of one price applies for households' nondurable goods. Defining $P_{i,t}(j) = \varepsilon_{i,t} P_{i,t}^i(j)$ for all countries. Where $\varepsilon_{i,t}$ is the nominal exchange rate and $P_{i,t}^i(j)$ is the price of differentiated goods in country i expressed in producer's currency. Substituting for $P_{i,t}(j)$ and log linearising around a steady state yields:

$$p_{f,t} = \int_0^1 \left(e_{i,t} + p_{i,t}^i\right) di = e_t + p_t^*, \tag{24}$$

where $e_t \equiv \int_0^1 e_{i,t} di$ is the log nominal effective exchange rate and $P_{i,t}^i \equiv \int_0^1 P_{i,t}^i(j)\, dj$ represents log domestic price index for country i.

Let real exchange with country i be $\wp_{I,t} \equiv \frac{e_{i,t} P_t^i}{P_t}$, and $e_t = \int_0^1 e_{f,t} df$ be the log effective real exchange rate. The real exchange rate is given by:

$$e_t = \left[\alpha \left(S_t\right)^{\eta-1} + (1-\alpha)\right]^{\frac{-1}{1-\eta}} = e\left(S_t\right), \tag{25}$$

where $e_{i,t} \equiv \log \wp_{i,t}$.

3.5 International risk sharing and uncovered interest rate parity

The model features an incomplete international markets. This implies that a condition analogous to Euler Eqs. 8 and 17 must not hold for households and entrepreneurs in any other country. Dividing the two equations yields an uncovered interest rate parity:

$$R_t^h = R_t^f \frac{\left[\lambda_t + \zeta^t E_t \left(\frac{C_{t+1}}{C_t}\right)\right]}{\left[\beta^t E_t \left(\frac{C_{t+1}}{C_t}\right)\right]}. \tag{26}$$

Eq. 26 is the arbitrage condition for holding domestic and foreign bonds.

3.6 Monetary policy

The monetary policy is conducted by means of a simple Taylor rule:

$$\frac{R_t^h}{R} = \left(\frac{R_{t-1}^h}{R}\right)^{\rho_r} \left[\left(\frac{\pi_t^h}{\overline{\pi}^h}\right)^{\rho_\pi} \left(\frac{Y_t^h}{\overline{Y}^h}\right)^{\rho_y}\right]^{1-\rho_r} \left(\frac{\epsilon_t}{\epsilon_{t-1}}\right)^{\frac{\rho_\epsilon}{1-\rho_\epsilon}}, \tag{27}$$

where R is the steady state real interest rate. ρ_r is interest rate smoothing - a backward looking parameter for interest rate gap. ρ_π is the weight the central bank assign to deviation of CPI inflation from the inflation target. ρ_y is weight on output gap. $\rho_\epsilon \succ 0$ is weight assigned to deviation of exchange rate for two subsequent periods. $\epsilon_{r,t}$ is policy shock that follows an AR (1) process, $\epsilon_{r,t} = \rho_r \epsilon_{r,t-1} + u_{1,t}$ with $u_{1,t} \sim i.i.d.\,(0, \sigma_\epsilon^2)$.

3.7 Market clearing conditions

The necessary market clearing conditions are as follows. For markets to clear in the domestic economy, it requires that domestic households and entrepreneurs' expenditure on durable goods must equal aggregated domestic production and costs associated with prevailing resources originating from price adjustments. Aggregate production in durable and nondurable sectors are equal in domestic and foreign economies respectively as follows;

$$A_t D_{t-1}^{\omega} N_t^{1-\omega} = \omega X_t + \frac{\upsilon_d}{2} \left(\pi_{d,t} - 1\right)^2, \tag{28}$$

$$A_t D_{t-1}^{\omega} N_t^{1-\omega} = (1-\omega) C_t \left[Y_{c,t}^h (1-\alpha) \left(\frac{P_t^h}{P_t}\right)^{\eta} + \alpha Y_{c,t}^f \left(\frac{P_t^f}{P_t}\right)^{\eta}\right] + \frac{\upsilon_c}{2} \left(\pi_{c,t} - 1\right)^2. \tag{29}$$

The debt market clears as follows.

$$(1-\alpha) B_t^h + \alpha B_t^f = 0. \tag{30}$$

The labour market clears as follows.

$$N_t = (1 - \alpha) N_t^h + \alpha N_t^f. \tag{31}$$

Eq. 31 is due to the assumption that labour is non-movable across countries.

Finally, fiscal authority do not issue transfers to mitigate economic fluctuations. Hence:

$$T_t = 0. \tag{32}$$

3.8 Deterministic steady state conditions

The deterministic steady state conditions are as follows. Inflation is zero in both durable and nondurable goods sectors. Shadow value of debt is always positive. This implies that entrepreneurs prefer to hold positive amounts of credit. Evaluating Eq. 17 using the standard steady state Lucas asset price equation $R = \beta^{-1}$ yields:

$$\psi = \left(\zeta^t - \beta^t\right) \succ 0. \tag{33}$$

Further, evaluating Eq. 18 in steady state combined with Eq. 33, domestic borrowers' consumption of durable goods is presented as;

$$\frac{D}{C} = \frac{\alpha}{1-\alpha} q \left\{ \left[1 - (1-\delta)\left(\beta + \chi\left(\beta - \zeta\right)\right)\right] \right\}^{-\eta}, \tag{34}$$

whereas $\delta \rightarrow 0$ (nondurability of goods) coupled by/or $\beta = \zeta$, (nonbinding collateral constraints, $\lambda = 0$) implies that q is the only determinant of the margin on durable /nondurable goods' consumption.

Notice that, collateral as a requirement for borrowing is iso-morphic to debt elastic interest rate. That is, an increase in the value of credit limit results to a decrease in demand for durable goods for collateral. Intuitively, as it becomes more difficult for domestic borrowers to convert collateral into new foreign debt, durable goods attractiveness as collateral diminishes.

4 Calibration

To solve the model, I linearise equations around their deterministic steady state. Log linear approximation of the market clearing conditions is then undertaken under the specifications $\psi_t = \left(\beta^t - \zeta^t\right) \succ 0$ and binding collateral constraint Eqs. 13 and 14.

I calibrate the model to South African economy using three approaches. Firstly, by computing averages of quarterly time series data for the period 1960Q01-2014Q02 obtained from South African Reserve Bank. Secondly, through assigning parameters median estimates from recent South African literature namely; Steinbach et al. (2009), Alpanda et al. (2010), Gupta and Steinbach (2010) and du Plessis et al. (2014a,b). Median values are chosen because they are not affected by outliers or skewness. Thirdly, by setting parameter values based on benchmark New Keynesian DSGE literature.

Computed parameter values are as follows. The quarterly aggregate durables depreciation rate δ is obtained from quarterly values of $\frac{D}{Y}$. This results in quarterly depreciation rate of 0.056. Quarterly share of durable consumption ω is calculated from quarterly values of $\frac{D}{C}$ yielding a ratio of 0.22. This is consistent with Hoosain (2012) that obtained a ratio of 0.2.

Parameters assigned from recent South African literature are as follows. Elasticity of substitution between domestic and foreign goods $\eta = 0.591$ as in Steinbach et al. (2009). Degree of openness, that is, import share of domestic consumption $\alpha = 0.28$ as in Alpanda et al. (2010). The value is 0.3 in empirical literature for developed countries. Elasticity of labour supply $\varphi = 3$ as in Steinbach et al. (2009) and Gupta and Steinbach (2010). Probability of resetting domestic price in a Calvo manner $\theta = 0.674$ as in du Plessis at al. (2014b). Elasticity of substitution between varieties of goods $\varepsilon_j = 6$ as in Alpanda et al. (2010) implying a steady state mark up $\mu = 0.2$. Durable consumption adjustment cost $\xi = 1.513$ as in du Plessis et al. (2014b). Monetary policy follows Taylor rule. That is, weight to deviation of CPI inflation from the inflation target is $\rho_\pi = 1.464$ as in Gupta and Steinbach (2010). This value is approximately 1.5 as used in standard Taylor rule. Weight on output gap $\rho_y = 0.294$ as in Alpanda et al. (2010) which is a slight variance from 0.5 in Taylor rule. Interest rate smoothing parameter $\rho_r = 0.83$ an in du Plessis et al. (2014a). This value is consisted with most international literature such as 0.8 in Merola (2009).

Parameters set following benchmark New Keynesian DSGE literature are as follows. An annual rate of return of 4 percent is underpinned by households' (savers) discount factor $\beta^t = 0.99$ implying that $\frac{1}{\beta^t} = 1.04$. I also set entrepreneurs' (borrowers) discount factor $\zeta^t = 0.98$ as estimated by Krusell and Smith (1998) and used in the literature by Monacelli (2009), Faia and Iliopulus (2009) and Iacoviello (2005). Elasticity of substitution between durable and nondurable goods is set $\varkappa = 0.2$[9].

[9] As highlighted in Monacelli (2009), footnote 14, the results reported do not depend on the value of the elasticity of substitution between durable and non-durable consumption. Monacelli (2009) and reference there in set this parameter equals to 1. I employ different values of between $0.2 - 0.9$ in the simulation to check for any variations in the results. This ensures deterministic steady state

Preference parameter is set $\nu = 0.3$ implying that South African households prefer to work $\frac{1}{3}$ of their time endowment. Country-specific risk premium is set $\phi = 0.0294$ as in international business cycle literature. Aswath Damodaran[10] computes this value to be 0.024 for South Africa. Share of durable goods used as domestic collateral $\sigma = 0.75$ following Iacoviello and Minetti (2006). Fraction of durable goods that cannot be used as collateral is set as $\chi = 0.2$ implying that on average, South African borrowers requires 20 percent down payment to obtain mortgage loans[11]. This yields a loan-to-value (LTV) ratio of 80 percent. The higher the LTV ratio the more binding collateral constraint become. This is higher that 0.70 used by Monacelli (2009). Iacoviello (2005) on the other hand sets a LTV ratio of 0.89. Finally I let shocks to the small open economy to be persistent at 0.9 with a standard deviation of 0.08 as in Faia and Iliopulus (2009) and Merola (2009). Table 1 summarises calibrated parameters.

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in the model set up. The simulation yields the same results.

[10]See http://pages.stern.nyu.edu/~adamodar/New_Home_Page/datafile/ctryprem.html.

[11]Fraction of durables that cannot be used as collateral is set 0.2 (20 percent) to ensure a loan-to-value ratio of 80 percent and guarantees binding collateral constraint. Adrian Goslett, CEO of RE/MAX of Southern Africa, suggests that in South Africa, deposit for home loans ranges between 10 − 30 percent. Higher percentage of the deposit, improves borrowers risk rating, lowers interest on the loan resulting in lower monthly loan repayment amount.

See http://www.property24.com/articles/deposit-helps-you-get-a-home-loan/15667 and http://www.property24.com/articles/bank-home-loans-and-deposits/15885.

Table 1: Parameter settings				
Parameter	**Description**	**BM model**	**CMI model**	
		Flexible price	**Sticky price**	**Flexible price**
β^t	Households' discount factor	0.99	0.99	0.99
ζ^t	Firms' discount factor	0.98	0.98	0.98
ν	Hours worked preference parameter	0.3	0.3	0.3
φ	Elasticity of labour supply	3	3	3
ω	Share of durable goods	0.22	0.22	0.22
$\varkappa$	Elasticity of substitution between durable and nondurable goods	0.2	0.2	0.2
α	Index of openness	0.28	0.28	0.28
η	Elasticity of substitution between domestic and foreign goods	0.591	0.591	0.591
ξ	Durables consumption adjustment cost	1.513	1.513	1.513
δ	Durable goods' depreciation rate	0.056	0.056	0.056
ε	Elasticity of substitution between variety of goods	6	6	6
ϕ	Country-specific risk premium parameter	0.0294	0.0294	0.0294
σ	Share of durable goods used as domestic collateral	-	0.75	0.75
χ	Fraction of durable goods' value that cannot be used as collateral	-	0.20	0.20
θ_c	Calvo probability of resetting price in nondurables sector	0.326	0.674	0.326
θ_d	Calvo probability of resetting price in durable sector	0.326	0.674	0.326
Notes: In the case of sticky nondurables, adjustment cost is set to zero while under sticky durables is 1.513.				

5 Results

This section evaluates the benchmark New-Keynesian DSGE model with perfect financial markets (BM hereafter) by determining the performance of the model in reaction to technology shock, monetary policy shock and country-specific risk premium shock. The focus is to determine the role of collateral constraint effects in determining the direction of movement of durables and nondurables in a small open economy. A comparison of the BM model, to an alternative, the credit market imperfections (CMI hereafter) model that incorporates collateral constraint is undertaken to present further understanding on the model's dynamics in explaining co-movement problem in durable and nondurable sectors. The two models are analysed with varying degree of price stickiness in the durable and nondurable sectors.

5.1 Co-movement problem under perfect financial markets

This section provides an analysis of the role of collateral constraint effect in determining the direction of movement of durables and nondurables in a small open economy facing perfect financial markets. It presents the behaviour of households and entrepreneurs in response to domestic and foreign technology, monetary policy and country-specific risk premium shocks under BM model. In the BM model, borrowing constraint is excluded from entrepreneurs optimization problem (i.e., $\psi = 0$). Fig. 2 represents the impulse response functions to shocks.

Fig. 2 displays the effect of selected variables to shocks in the BM model. The restriction cases are illustrated as: sticky nondurable and flexible durable prices, sticky durable and flexible nondurable prices and equally sticky durable and nondurable prices. A twofold observations emerges. First, the flexibility of prices of durables does not govern the response of aggregate consumption to shocks. This is due to the fact that entrepreneurs face constant returns to scale and there are no barriers to moving factors of production across sectors. Consequently, temporary shocks are neutral to aggregate consumption of nondurables and durable purchases. Second, existence of sticky durable prices is the least required for attaining positive co-movement in durable and nondurable sectors. In all the cases, it is evident that collateral constraint plays a critical role in escalating and spreading shocks in the domestic economy, however, co-movement problem do not arise.

Particularly, in response to a contractionary monetary policy shock, an increase in nominal short-term interest rate causes the exchange rate to depreciate thereby resulting in a fall in aggregate demand for durables and nondurables.[12] This has four stylised effects namely: a rise in the cost of durables as inputs for production, a decrease in the shadow value of domestic and foreign borrowing constraint occasioned by disincentive for entrepreneurs to borrow at high interest rate, a rise in user cost of durables due to diminishing discounted future value of durables, and an increase in interest burden on domestic and foreign borrowing thereby resulting in a decrease in the demand for durables. Reduced demand for durables causes its relative price to decline in the domestic economy. As a result, from the findings, it can be concluded that responses of durable and nondurable consumption to shocks in the small open

[12] That is, in the face of technology shock, productivity of labour and durable goods rises for all entrepreneurs resulting in a decline in the marginal cost and aggregate consumption of non-durables. The exchange rate depreciates causing substitution of foreign produced goods for domestic good – an increase in exports compared to imports. This in turn yields improved foreign asset position for the domestic economy leading to a decrease in the country's risk premium and the interest rate. The monetary policy shock in turn causes an increase in aggregate consumption of nondurables and durable purchases.

economy set up does not reveal the co-movement problem reported in closed economy literature.

The insight of this incongruity is the unique element of durable goods in perfect financial markets. Namely, the shadow value of durable goods, λ_t - is the product of the relative price of durables and the marginal benefit of purchasing durables - is approximately constant.

$$\lambda_t \equiv \frac{q_t}{C_t} = E_t \left\{ \sum_{j=0}^{\infty} \frac{\zeta^t}{C_{t+1}} \left(\varpi \frac{Y_{t+1}}{D_t} + q_{t+1} \right) \right\} \simeq const.$$

The intuition to this constancy can be traced to typically high stock-to-flow ratio of durable goods. This is due to firstly, the marginal benefit of durables is dependent on stock of durables. Secondly, given constant returns to scale in production ($\varpi < 0$), the elasticity of substitution between durables and nondurables is almost infinite. Specifically, additional purchase of durable inputs by entrepreneurs, due to the responsiveness to its own user cost, does not improve the total utility. In addition, neutrality of temporary shocks is at the centre of irrelevance of price stickiness in the BM model. That is, in the face of shocks, entrepreneurs sustain demand of durables at predetermined prices and marginal cost of additional units of j goods is $\frac{\nu N_t}{MP_{j,t}^N}$,where $MP_{j,t}^N$ is the marginal product of labour. The initial increase in aggregate output in the sticky price sector increases the demand for labour thus increasing νN_t and inflates marginal cost. Due to increasing marginal costs, by increasing prices, flexibly priced entrepreneurs can uphold their markup (Barsky et al., 2003).

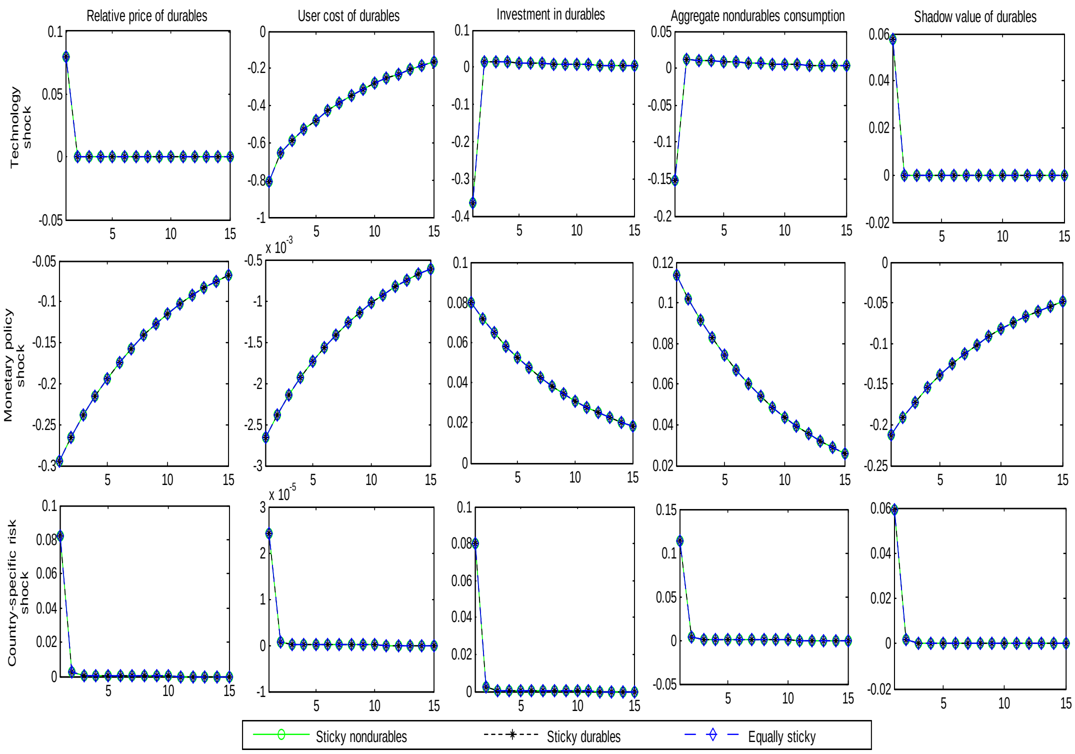

Figure 2: IRFs to positive shocks in the BM model

5.2 Role of credit market imperfections

This section provides an analysis of the role of collateral constraint effect in determining the direction of movement of durables and nondurables in a small open economy facing imperfect financial markets. In the CMI model, entrepreneurs face domestic and foreign borrowing constraints and set $\psi \succ 1$. Fig. 3 represent the impulse response functions of selected variables to shocks in the CMI model.

It is noticeable that collateral constraint plays a crucial role in determining the impact of monetary policy shock on durable and nondurable consumption. According to Fig. 3, responses to contractionary monetary policy shock are at odds with Monacelli (2009). For instance, in the face of a positive monetary policy shock, demand for durables and nondurables ought to decline that in turn increases the relative price of durables. However, responses of durable purchases and nondurable consumption display positive co-movement in CMI models. This is due the presence of both domestic and foreign collateral constraints. That is, from the shadow value of durables $\lambda_t^h \chi_h \sigma q_{t+1} + \lambda_t^f \left(1 - \sigma\right) q_{t+1} \widetilde{\chi}_{t+1}^f$in Eq. 20, it is evident that marginal benefit of durable purchases by entrepreneurs depend on the proportion of domestic LTV versus foreign LTV and whether the borrowing constraints are binding or not. In our case, domestic LTV $\sigma = 0.80$ therefore, as entrepreneurs reallocate purchase of durables from foreign to domestic, the collateral constraint is relaxed and therefore the CMI model reverts to a BM model.

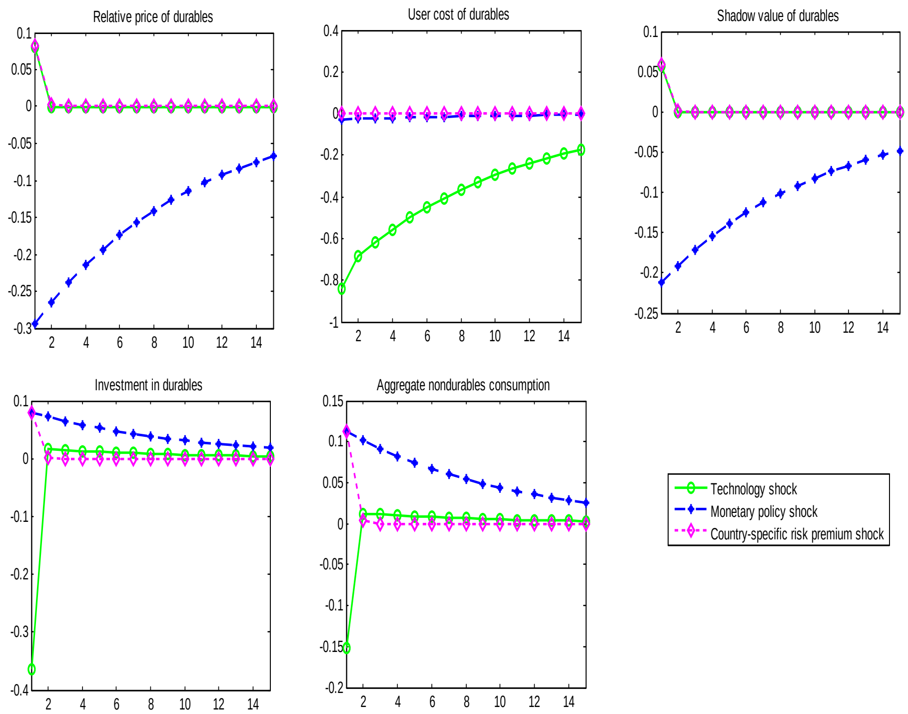

Figure 3: IRFs to positive shocks in the CMI model

6 Conclusion

This paper has presented a New Keynesian DSGE small open economy model with collateral constraint in determining the direction of movement of durable and nondurable goods. The small open economy is characterised by varying perfect and imperfect financial markets and degree of price stickiness simulated to South African data over quarterly time frequency. The analysis suggests that collateral constraint escalates and spreads shocks to the domestic economy. Collateral constraint effects therefore plays a crucial role in determining the impact of shocks on durable and nondurable consumption. Responses of durable and nondurable consumption to shocks in the small open economy set up does not reveal the co-movement problem reported in closed economy literature. Flexibility of prices of durables and nondurables do not govern the response of aggregate output in the face of shocks. The results reveal that collateralized borrowing help rectify the inconsistency caused by agency problem in credit markets through the net worth of households and entrepreneurs. Therefore, a policy intervention that sets the minimum LTV ratio on borrowing sustains accumulation of durable goods required for production by entrepreneurs.

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Appendix

This appendix recaps the full nonlinear model. The benchmark model (BM) consists of 30 endogenous variables and 4 exogenous AR(1) stochastic shock process. To build the credit market imperfections (CMI) model from the BM model, 4 endogenous variables are added to the BM model as follows. Firstly, Eqs. 12, 13, 16, 17 and 18 above are added to the CMI model that includes the following 5 endogenous variables: B_t^f, ψ_t, λ_t^h, λ_t^f and $\widetilde{\chi}_{t+1}^f$. Secondly σ is set to 0.75 and $\chi = 0.25$. The 30 endogenous variables in the BM model are as follows.
$W_t, N_t, C_t, C_t^h, C_{l,t}, D_t, \widetilde{D}_t, D_t^{Share}, D_t^{Shadow}, X_t, Z_t, B_t^h, R_t^h, R_t^f, Q_t, P_t^h, P_t^f, FA_t, E_t, S_t, A_t, \pi_{c,t}, \pi_t^d, \pi_t^c, \epsilon_t^r, \epsilon_t^{risk}, \tau_t, Y_t, Y_t^{total}, \widehat{MC}$.

These variables jointly with the additional variables in the CMI model solve the following 35 equations.

Households:

$$W_t = \frac{\nu}{\left[(1-\alpha)^{\frac{1}{\eta}} + \alpha^{\frac{1}{\eta}}\right]\left[(1-\omega)^{\frac{1}{1-\varkappa}}\right]} C_t N_t^{\varphi}, \tag{A.1}$$

$$\frac{1}{C_t} = \beta^t E_t\left(\frac{R_t^h}{C_{t+1}}\right), \tag{A.2}$$

$$\frac{q_t}{C_t} = \frac{j}{D_t} + \beta^t E_t\left(\frac{q_{t+1}}{C_{t+1}}\right), \tag{A.3}$$

$$C_t + q_t X_t + B_t^h + e_t B_t^f = R_{t-1}^h B_{t-1}^h + \Xi\left(FA_{t-1}, \varepsilon_{t-1}^{risk}\right) e_t R_{t-1}^f \frac{e_t}{e_{t-1}} B_{t-1}^f + W_t N_t + T_t. \tag{A.4}$$

Entrepreneurs:

$$Y_t = A_t D_{t-1}^{\varpi} N_t^{1-\varpi}, \tag{A.5}$$

$$R_t^h B_t^h \leq E_t\left(\chi_h \sigma q_{t+1} D_t\right), \tag{A.6}$$

$$R_t^f B_t^f \leq E_t\left[q_{t+1}\left(1-\sigma\right) D_t \left(1 - \frac{1-\chi_f}{qd}\left[q_{t+1}\left(1-\sigma\right) D_t\right]\right)\right], \tag{A.7}$$

$$\pi_{j,t}^h = \zeta^t E_t\left(\pi_{j,t+1}^h\right) + \frac{\varepsilon_j - 1}{\upsilon_j} \widehat{mc}_{j,t}, \tag{A.8}$$

$$W_t = \left(1-\varpi\right) \frac{Y_t}{N_t}, \tag{A.9}$$

$$\frac{1}{C_t} = E_t\left(\frac{\zeta^t R_t^h}{C_{t+1}}\right) + \lambda_t^h R_t^h, \tag{A.10}$$

$$\frac{1}{C_t} = E_t\left(\frac{\zeta^t R_t^f}{C_{t+1}}\right) + \lambda_t^f R_t^f \psi_t, \tag{A.11}$$

$$Z_t \equiv \frac{1}{q_t}\left\{E_t \frac{\zeta^t}{C_{t+1}}\left(\varpi Y_{t+1} + D_t q_{t+1}\right) + D_t E_t\left[\lambda_t^h \chi_h \sigma q_{t+1} + \lambda_t^f \left(1-\sigma\right) q_{t+1} \widetilde{\chi}_{t+1}^f\right]\right\}. \tag{A.12}$$

Market clearing conditions:

$$A_t D_{t-1}^{\varpi} N_t^{1-\varpi} = \omega X_t + \frac{\upsilon_d}{2}\left(\pi_{d,t} - 1\right)^2, \tag{A.13}$$

$$A_t D_{t-1}^{\omega} N_t^{1-\omega} = \left(1-\omega\right) C_t \left[Y_{c,t}^h \left(1-\alpha\right)\left(\frac{P_t^h}{P_t}\right)^{\eta} + \alpha Y_{c,t}^f \left(\frac{P_t^f}{P_t}\right)^{\eta}\right] + \frac{\upsilon_c}{2}\left(\pi_{c,t} - 1\right)^2, \tag{A.14}$$

$$\left(1-\alpha\right) B_t^h + \alpha B_t^f = 0, \tag{A.15}$$

$$Y_{c,t} + Y_{d,t} = N_t = 0, \tag{A.16}$$

$$T_t = 0, \tag{A.17}$$

$$e_t = 0. \tag{A.18}$$

Monetary policy:

$$\frac{R_t^h}{R} = \left(\frac{R_{t-1}^h}{R}\right)^{\rho_r} \left[\left(\frac{\pi_t^h}{\overline{\pi}^h}\right)^{\rho_\pi} \left(\frac{Y_t^h}{\overline{Y}^h}\right)^{\rho_y}\right]^{1-\rho_r} \left(\frac{\epsilon_t}{\epsilon_{t-1}}\right)^{\frac{\rho_\epsilon}{1-\rho_\epsilon}}. \tag{A.19}$$

Definitions:

$$C_{l,t} = \left[(1-\omega)^{\frac{1}{\varkappa}} C_t^{\frac{\varkappa-1}{\varkappa}} + \omega^{\frac{1}{\varkappa}} \widetilde{D}_t^{\frac{\varkappa-1}{\varkappa}}\right]^{\frac{\varkappa}{\varkappa-1}}, \tag{A.20}$$

$$\widetilde{D}_t \equiv \left(D_{t-1} - \frac{\xi}{2} \frac{\left(X_t - \delta D_{t-1}\right)^2}{D_{t-1}}\right), \tag{A.21}$$

$$X_t = D_t - (1-\delta) D_{t-1}, \tag{A.22}$$

$$\widetilde{\chi}_{t+1}^f \equiv \left[1 - 2\left(1-\chi_f\right)(1-\sigma) \frac{q_{t+1} D_t}{qd}\right], \tag{A.23}$$

$$D_t^{Share} = (1-\omega) C_t, \tag{A.24}$$

$$D_t^{Shadow} = (1-\alpha) \frac{q_t}{C_t}, \tag{A.25}$$

$$Y_t = \omega N_t + (1-\omega) N_t. \tag{A.26}$$

Small open economy dynamics:

$$S_{i,t} = \frac{P_t^f}{P_t^h}, \tag{A.27}$$

$$P_t \equiv \left[(1-\alpha) P_{h,t}^{1-\eta} + \alpha P_{f,t}^{1-\eta}\right]^{\frac{1}{1-\eta}}, \tag{A.28}$$

$$\pi_t = \pi_{h,t} \frac{\left(\alpha + (1-\alpha) S_t^{1-\eta}\right)^{\frac{1}{1-\eta}}}{\left(\alpha + (1-\alpha) S_{t-1}^{1-\eta}\right)^{\frac{1}{1-\eta}}}, \tag{A.29}$$

$$e_t = \left[\alpha \left(S_t\right)^{\eta-1} + (1-\alpha)\right]^{\frac{-1}{1-\eta}}, \tag{A.30}$$

$$R_t^h = R_t^f \frac{\left[\lambda_t + \zeta^t E_t \left(\frac{C_{t+1}}{C_t}\right)\right]}{\left[\beta^t E_t \left(\frac{C_{t+1}}{C_t}\right)\right]}. \tag{A.31}$$

Exogenous AR(1) shock process:

$$a_t = \rho_a a_{t-1} + \epsilon_{a,t}, \tag{A.32}$$

$$\epsilon_{r,t} = \rho_r \epsilon_{r,t-1} + u_{1,t}, \tag{A.33}$$

$$\varepsilon_t^{risk} = \rho_{risk} \varepsilon_{t-1}^{risk} + u_t^{risk}. \tag{A.34}$$