Redistributive policies and growth-inequality nexus

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Abstract

The paper studies the inequality-growth nexus when various fiscal policies are involved. We find that the relationship between before-tax and -transfer inequality and after-tax and -transfer inequality ("market" and "net inequality", respectively), and growth could be different. In the short-run, only market inequality is important for growth whereas both are important in the long run. Two main channels are identified through which fiscal policy could have influence on the inequality-growth nexus: a direct income effect on the post-tax and -transfer income distribution and an indirect substitution effect on the pre-tax and -transfer income distribution. In ranking various policies based on their relative impacts on inequality, consumption tax is found to have the least impact as it merely influence inequality through the substitution effect whereas income tax has a relatively stronger impact on inequality through both effects.

1. Introduction

During the last three decades, inequality has risen in most countries in the world, in many of them, there is a steep increase. Cornia and Court (2001) document, over the two decades since the 1980s, income inequality has risen in more than two thirds of the countries in the world for which 'high quality data' is available. Between 1990 and 2010, according to the IMF (Ostry et al., 2014), the GINI for disposable income has increased in nearly all advanced economies and emerging European economies, most economies

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in Asia and the Pacific, and the Middle East and North Africa. In Africa and Latin America, inequality has increased in one-fourth and one-third of the economies, respectively, although there was a slight decline in average inequality. The extent of rising inequality has been a subject of concern amongst academics and policymakers, as it threatens growth sustainability and poverty reduction strategies.

Economists have identified a number of channels through which inequality could harm growth. Galor and Zeira (1993), Benabou (2000, 2002[7]) and Getachew (2010, 2012), among others, show that in the face of capital market imperfections, inequality has a definite negative effect on growth, because relatively more high-return investment opportunities would be forgone by resource poor households in inegalitarian societies than egalitarian ones. From a political economy context, Alesina and Rodrik (1994) and Persson and Tabellini (1994) argue that inequality is harmful for growth, because it demands large transfers of income from the rich to the poor that distorts saving, resulting in lower capital investment and, hence, lower growth. Inequality could also discourage growth by increasing socio-political instability, which, in turn, decreases investment (e.g., Alesina and Perotti, 1996 and Benhabib and Rustichini, 1996). Such theories are often supported by empirical findings (see, for instance, Ostry et al. 2014).[1]

Public spending, whether in the form of public investment or transfer is usually justified as a means for redistribution. During most of the 1980s and the 1990s, government expenditure has mainly trended upwards. Total government expenditure increased in many developing countries, although there has been a slight decline in the government spending to GDP ratio. Fan and Rao (2003) report that during recent periods government spending in 43 developing countries averaged annual growth rates of 4% and 5.7%. The trend in public capital stocks and flows specifically are also increasing. Between 1960 and 2000, for instance, the public capital stock in many of the OECD and non-OECD countries grew at an average rate of 3.3% and 5.7%, respectively (Arslanalp et al., 2010). Such a marked rise in public spending in the face of a rising inequality emphasizes the need for a well specified analytical framework, within which the relationship between public policy

[1]High inequality could also stifle efforts to reduce absolute poverty, since the poor gain less in absolute terms from growth (Ravallion, 1997). Using data for African economies, Fosu (2008, 2009) shows that decreasing inequality mainly tends to reduce poverty.

choices and the inequality-growth nexus is systematically addressed.

In this paper we analyze the mechanism through which fiscal policy has effect on inequality-growth trade-offs. To do so we develop a growth model where both growth and inequality are endogenously determined. In the model, individuals are different in terms of their initial capital wealth. The credit markets are incomplete. Firm level production takes place using a constant elasticity of substitution production function with public and private capitals. The government uses consumption and income taxes to finance productive government expenditure and a transfer program. The government also redistributes income through a negative income tax program. The source of endogenous growth is public capital combined with private capital in a constant return to scale production function that makes the cumulative marginal product of capital in the long-run constant (Barro, 1990). Endogenous inequality is rather generated due to the credit market imperfection, as in Loury (1981).

When individuals are not allowed to make unlimited borrowing and lending, mobility and inequality persist. The dynamics of aggregate variables – public and private capital, and the growth rate of the economy – and inequality are jointly determined. Diminishing returns to investment imply certain individuals, in particular, the poor have a higher marginal product than the rich. But, since they cannot borrow and invest efficiently due to credit constraints, Pareto efficiency cannot be achieved as often implicitly assumed in representative-agent models with complete markets. Therefore, a higher inequality leads to a greater inefficiency. When the equity efficiency trade-off is mitigated in this manner, redistributive public policies could have additional efficiency effects.

Two main channels are identified through which fiscal policy could have influence on the inequality-growth nexus. The first is a direct *income effect* on the post-tax and -transfer income distribution. The second is an indirect *substitution effect* on the private-public capital ratio that influences the pre-tax and -transfer income and determines the distribution of the next period investment and income. The key mechanism generating the substitution effect is the substitutability-complementarity relationship between private and public capital in production. Substitutable public investment could provide poor households the opportunity to relax credit constraints that impede their investment through factor substitution. Complementary public investment, on the other hand, could rather benefit disproportionately the rich, who own much of the private resource in the economy.

An increase in income tax or transfer decreases the post-tax income inequality whereas a change in consumption tax has no impact on the post-tax income. However, an increase in consumption tax could mitigate inequality through decreasing the private-public capital ratio if the elasticity of substitution between the factors is less than unity, and conversely. An increase in transfer, on the other hand, increases the private-public capital ratio, as it diverts resource away from public investment. This could have the "wrong" effects on inequality if the two inputs are more of substitutes; however, such a substitution effect could be partly offset through an income effect of decreasing the post tax income inequality. Similarly, income tax has both substitution and income effects on the post tax income distribution.

Ranking various policies based on their relative impacts on the distributional dynamics, consumption tax has the least impact on inequality, as it merely influence inequality through the substitution effect. In contrast, income tax and transfer have a relatively stronger impact on inequality through both the substitution and income effects. Among the latter two, income tax has a stronger effect, due to its distortionary effect. A higher income tax not only increases public capital but also decreases private capital whereas consumption tax or transfer have no such distortionary impact.

By considering a unified framework, the paper contributes to a variety of strands in the literature related to: infrastructure and growth (e.g., Barro, 1990, Futagami et al., 1993, Agenor, 2008); inequality and growth (e.g., Alesina and Rodrik, 1994, Persson and Tabellini, 1994, Galor and Zeira, 1993 and Benabou, 1996; 2000; 2002); and public capital and income inequality (see, for instance, Lopez, 2003, Calderon and Servén, 2004, García-Peñalosa and Turnovsky, 2007, Chatterjee and Turnovsky, 2012, and Getachew, 2010, 2012, and Getachew and Turnovsky, 2014). It extends the literature by considering the joint allocation of public expenditure and identifying new channels, through which, that allocation can affect inequality-growth tradeoffs.

2. The Model

2.1. Preference and technology

Suppose a continuum of overlapping generation households, $i \in (0,1)$. The ith household initially endowed with k_{i0} units of private capital and g_0 units of a non-rival and a non-excludable public productive input.

Agents maximize their utility in accordance to the utility function:

$$U_i \equiv \max_{\{c_{it},k_{it+1}\}} (1-\beta)\ln c_{it} + \beta \ln k_{it+1} \tag{1}$$

subject to the budget constraints

$$c_{it}(1+\tau_c) + s_{it} = (1-\tau_y)\, y_{it} + T_t \tag{2}$$

Each household owns a firm that operates with a CES production technology:

$$y_{it} = a\epsilon_{it}\left(\alpha (g_t)^\rho + (1-\alpha)(k_{it})^\rho\right)^{1/\rho} \tag{3}$$

which implies that the credit market is imperfect, in the spirit of Benabou (2002). c_{it} and y_{it} stand for individual consumption and income, respectively;[2] ϵ_{it} represents the idiosyncratic shocks. Individual capital endowment, k_{it}, is assumed to be lognormally distributed: $\ln k_{i0} \sim N\left(\mu_0, \sigma^2_{0,k}\right)$. We also assume $\ln \epsilon^i_t \sim N\left(-\upsilon^2/2, \upsilon^2\right)$. g_t is a public input, at time t, which is identical among households. Each household supplies a unit of labour inelasticly.

The policy parameters T_t, τ_y and τ_c denote money transfer, flat rate income and consumption taxes, respectively. We let the government to hold a constant fraction of transfer at all time, which is a necessary condition for a balanced growth path. In this case, $T_t \equiv my_t$ is the break-even income where m is the fraction of transfer over aggregate income. Only individuals with post-tax income lower than my_t qualify for the negative income tax (NIT).[3] The net amount that each individual *receives* is given by:

$$\max\left((my_t - (1-\tau_y)\, y_{it} - \tau_c c_{it}), 0\right) \tag{4}$$

where

$$\rho \le 1,\ \alpha \in (0,1),\ a > 0, \{\tau_c, \tau_y, m\} \in (0,1) \tag{5}$$

We suppose both public and private capitals to be fully depreciated, and, a constant population growth:

[2] Note that, variables with(out) superscript i represent individual (aggregate) variables.

[3] The choice of y_t as a threshold point is merely for simplicity. Rather, in general, $m\vartheta y_t$ could be considered as the break-even income level where $\vartheta > 0$. Therefore, the threshold point can be associated to any individual in the economy. We consider the case $\vartheta = 1$ here, with no loss of generality.

$$k_{it+1} = s_{it} \tag{6}$$

2.1.1. The government budget:

The government uses consumption and income taxes for financing public investment and transfer:

$$g_{t+1} + my_t = \tau_c c_t + \tau_y y_t \tag{7}$$

where $c_t \equiv \int_i c_{it}$ and $y_t \equiv \int_i y_{it}$, which implies that the government budget is balanced.

2.1.2. Households optimal decision

The ith agent optimal saving and consumption are thus given by:

$$s_{it} = \widetilde{y}_{it}\beta \tag{8a}$$

$$(1+\tau_c)\, c_{it} = \widetilde{y}_{it}\,(1-\beta) \tag{8b}$$

$$\widetilde{y}_{it} \equiv ((1-\tau_y)\, y_{it} + my_t) \tag{8c}$$

where $\widetilde{y}_{it}$ represents the individual disposable income.

Household consumption and saving increase with transfer but they decrease in income tax, whereas consumption tax has no effect on individuals' saving rate.

3. Aggregation and distributional dynamics

3.1. Aggregate consistency

Aggregate investment, consumption and income are given by, respectively:

$$k_{t+1} = (1-\tau_y+m)\,\beta y_t \tag{9a}$$

$$(1+\tau_c)\, c_t = (1-\beta)\,(1-\tau_y+m)\, y_t \tag{9b}$$

$$y_t = c_t + k_{t+1} + g_{t+1} \tag{9c}$$

Condition (9a) represents the aggregate capital investment at $t+1$, considering the capital market clearing condition $k_{t+1} = s_t$. Eq. (9b) is aggregate consumption as a fraction of disposable income. Combining (7), (9a) and

(9b), one gets (9c) which gives the economywide budget constraint at equilibrium. Therefore the economy's output is allocated between current consumption, public and private investment at all time. With full depreciation of capital in the economy, the latter also represent the next period capital stocks of the economy.

Note also that given a constant m, τ_y, and τ_c, the fraction of GDP that goes to infrastructure is always constant, insuring a balanced growth path. This is easily seen, from (9) where

$$g_{t+1} = \psi y_t \tag{10a}$$

$$\psi \equiv 1 - \frac{(1 - \tau_y + m)(1 + \beta\tau_c)}{1 + \tau_c} \tag{10b}$$

Considering the restrictions in (5), it is easy to see $\psi < 1$. However, m should be sufficiently small relative to τ_y for ψ to be positive. Therefore the following additional restriction in the policy parameters is required to guarantee a positive level of public investment,

$$m - \tau_y < \frac{\tau_c(1 - \beta)}{\beta\tau_c + 1} \tag{11}$$

which is the necessary and sufficient condition for $\psi > 0$. That means if $\tau_c = 0$, the government expenditure on transfer should be less than its revenue from income tax. But with additional revenue from the consumption tax, the government transfer could exceed the income tax as long as the transfer rate is kept sufficiently small (vis-a-vis τ_y and τ_c).

3.2. Aggregate variable ratios

Unlike most Ak models, the current model displays transitional dynamics, a feature attributed to existence of heterogeneity and market imperfection in the economy. In the model, the aggregate public and private capitals grow at the same rate at all time whereas the aggregate consumption and income grow at a similar rate. However, the growth rate of capital is only identical to a period-lag of consumption and income.

From (9a) and (10a), the aggregate capital ratio, is given by,

$$\varphi = \frac{\tau_c(1 - \beta)}{(1 + \tau_c)\beta} + \frac{\tau_y - m}{(1 - \tau_y + m)\beta} = \frac{\psi}{(1 - \tau_y + m)\beta} \tag{12}$$

where $\varphi \equiv g/k$, which is constant. The public-private capital ratio (φ) increases in τ_c and τ_y but decreases in β and m. Since the taxes are used to finance the public capital, their relations with φ are proportional. The more patient the households are (higher β) the higher the private saving and investment are and, hence, the lower φ becomes. Through depleting the available fund for public investment, the redistributive variable m decreases φ.

The aggregate consumption-capital ratio is derived from (9a) and (9b)

$$\frac{c_{t-1}}{k_t} = \frac{1}{1+\tau_c}\frac{1-\beta}{\beta} \tag{13}$$

Also, from (9a), one easily computes the aggregate income-capital ratio:

$$\frac{y_{t-1}}{k_t} = \frac{1}{(1-\tau_y+m)\,\beta} \tag{14}$$

Therefore, at the aggregate level, public and private capital (g_t and k_t, respectively) grow at the same rate at all times, which is similar to the growth rate of lag consumption (c_{t-1}) and income (y_{t-1}). As we see in the next section, such a feature exists despite the Ak setup due to existence of inequality dynamics.

3.3. Aggregate and distributional capital dynamics

The Appendix derives the dynamic system that characterize the path of the economy. The aggregate income of the economy is given by,[4]

$$y_t = g_t \alpha^{\frac{1}{\rho}} \left(x_t+1\right)^{\frac{1}{\rho}} z_t^{(1-\rho)/\left(2\rho^2\right)} \tag{15}$$

where

$$x_t \equiv (1-\alpha)\,\alpha^{-1}\varphi^{-\rho} e^{0.5\sigma_{t,k}^2\rho(\rho-1)} \tag{16a}$$

$$z_t \equiv \frac{x_t^2}{\left(x_t+1\right)^2}\left(e^{\rho^2\sigma_{t,k}^2}-1\right)+1 \tag{16b}$$

[4]See Appendix A for details in the derivation.

The term $z_t \geq 0$ in (15) arises due to $\sigma_{t,k}^2 \neq 0$ and will not occur in a representative economy. If $\sigma_{t,k}^2 = 0$, the economy's aggregate output (y_t^r) is given by: $y_t^r = \alpha^{\frac{1}{\rho}} \left(\left(1-\alpha\right)\varphi^{-\rho}+1\right)^{\frac{1}{\rho}}$.

Combining (9a) and (15) gives the dynamics of aggregate capital:

$$k_{t+1} = \left(1-\tau_y+m\right)\beta g_t \alpha^{\frac{1}{\rho}} \left(x_t+1\right)^{\frac{1}{\rho}} z_t^{(1-\rho)/\left(2\rho^2\right)} \tag{17}$$

Income tax has the standard distortional effects on the aggregate private capital accumulation. Since individual saving and investment increase in the transfer rate, aggregate private capital also increases in m. Of course, such impact will be compromised due to a trade off in g_t. But, consumption tax has no direct impact on aggregate private capital. This is intuitive given its neutral role on private investment and saving (8a).

And, from (10) and (15), one gets the dynamics for public capital:

$$g_{t+1} = \psi g_t \alpha^{\frac{1}{\rho}} \left(x_t+1\right)^{\frac{1}{\rho}} z_t^{(1-\rho)/\left(2\rho^2\right)} \tag{18}$$

All taxes increase the accumulation of the public capital as they are used to finance public investment. However, ψ and thus public capital accumulation decreases in m since the public expenditure is budget-neutral.

Finally, the dynamics of inequality is given by (see the Appendix)

$$\sigma_{t+1,k}^2 = \ln\left(s^2 z_t^{1/\rho^2} + 1 - s^2\right) \tag{19}$$

$$s \equiv \frac{1-\tau_y}{1-\tau_y+m} \tag{20}$$

Therefore the distributional dynamics (19), combined with capital dynamics, (17) and (18), characterize the transitional dynamics of the economy.

4. Consumption, income and capital inequality

Appendix A also derives the distributions of k_{it+1}, c_{it}, and y_{it}, respectively:

$$\sigma_{t+1,k}^2 = \sigma_{t,c}^2 = \underbrace{\ln\left(s^2\left(e^{\sigma_{t,y}^2}-1\right)+1\right)}_{\widetilde{\sigma}_{t,y}^2} \tag{21}$$

where

$$\sigma_{t,y}^2 = \rho^{-2} \ln \left(\frac{x_t^2}{(x_t+1)^2} \left(e^{\rho^2 \sigma_{t,k}^2} - 1 \right) + 1 \right) + \upsilon^2 \tag{22}$$

Therefore, considering $s < 1$, the relation between the dynamics of capital, consumption and income, is as follows:

$$\sigma_{t+1,k}^2 = \sigma_{t,c}^2 < \sigma_{t,y}^2 \tag{23}$$

The distribution of the next-period capital ($\sigma_{t+1,k}^2$) is similar to that of the current consumption distribution ($\sigma_{t,c}^2$) and the distribution of post-tax and -transfer income ($\widetilde{\sigma}_{t,y}^2$) but it is greater than the current pre-tax and -transfer income distribution ($\sigma_{t,y}^2$) due to the NIT program ($m \neq 0$). It is intuitive that next-period capital investment and current consumption have similar distribution. Current individual saving is the next-period capital investment due to complete depreciation and zero adjustment cost of capital in the economy. Since both consumption and saving are linear functions of individual disposable income, the two have similar distribution whereas the former determines the dynamics of capital.

Proposition 1. *With no lump sump transfer, the distributions of investment, consumption and income are equal regardless of the level of the tax rates. Otherwise, the relationship is given by (23).*

Proof. From (21), if $m = 0$, then $s = 1$ and $\sigma_{t+1,k}^2 = \sigma_{t,c}^2 = \sigma_{t,y}^2$. ■

5. Steady-state

From (19), the dynamics of inequality is history dependent. However, this is not the case for the dynamics of public and private capital, which rather depends on the dynamics of inequality. Because inequality is the driving force of transitional dynamics in the economy, it also determines the long-run property of the model. In particular, once the dynamics of inequality reaches its long-run equilibrium, the whole system will be in equilibrium.

Steady state distribution is given by

$$\sigma_k^2 = \ln \left(s^2 \left(e^{\sigma_y^2} - 1 \right) + 1 \right) \tag{24a}$$

$$\sigma_y^2 = \rho^{-2} \ln \left(\frac{x^2}{(x+1)^2} \left(e^{\rho^2 \sigma_k^2} - 1 \right) + 1 \right) + \upsilon^2 \tag{24b}$$

$$x \equiv (1-\alpha)\, \alpha^{-1} \varphi^{-\rho} e^{0.5 \sigma_k^2 \rho (\rho - 1)} \tag{24c}$$

5.1. Local stability

From (21) and (22), the condition for stability of the system can easily be derived:

$$\frac{\partial \sigma_{t+1,k}^2}{\partial \sigma_{t,k}^2} = \frac{\partial \widetilde{\sigma}_{t,y}^2}{\partial \sigma_{t,y}^2} D = s^2 \frac{e^{\sigma_{t,y}^2}}{e^{\widetilde{\sigma}_{t,y}^2}} D \approx D \tag{25}$$

where

$$D \equiv \frac{1}{\rho^2} \frac{x^2}{(x+1)^2} \left[\frac{1}{x^2/(x+1)^2 \left(e^{\rho^2 \sigma_k^2} - 1\right) + 1} \right] \left(\frac{\rho(\rho-1)}{1+x} \left(e^{\rho^2\sigma_k^2} - 1\right) + \rho^2 e^{\rho^2 \sigma_k^2} \right)$$

$D < 1$ is thus the sufficient condition for stability of the dynamics.

The second and last terms of (25) imply that a unit change in pre-tax and -transfer income distribution leads to a similar change in the post-tax and -transfer income distribution.

6. Inequality dynamics and fiscal policy: two channels

6.1. Income and substitution effects

What channels are there for policy to affect inequality dynamics? Given (21) and (20), in general, the inequality dynamics can be expressed as a function of policy variables:

$$\sigma_{t+1,k}^2 = \widetilde{\sigma}_{t,y}^2 = f\left(\sigma_{t,y}^2, s(\tau_y, m)\right) \tag{26}$$

where that $\sigma_{t,y}^2$ is the pre-tax and -transfer income distribution whereas τ_y and m denote the income tax and transfer rate, respectively. Given (22), $\sigma_{t,y}^2$ is determined by the policy parameters and the level of the distribution of capital in the current period:

$$\sigma_{t,y}^2 = q\left(\sigma_{t,k}^2, \varphi(\tau_c, \tau_y, m, \beta), \iota\right) \tag{27}$$

where $\iota \equiv \iota(\upsilon^2, \alpha, \rho)$.

Combining the above two gives,

$$\sigma_{t+1,k}^2 = h\left(\sigma_{t,k}^2, \varphi(\tau_c, \tau_y, m), s(\tau_y, m)\right) \tag{28}$$

which shows that wealth inequality at $t+1$ is determined by inequality at time t, ι and two policy variables φ and s; whereas $\sigma_{t,k}^2$ is pre-determined as capital is installed a period earlier, and ι denotes structural parameters.

From eqs. (26) to (28), there are two main channels through which policy affects the dynamics of inequality. The first is a direct *income effect* on the post-tax and -transfer income distribution. This is captured in (26), where a change in s (due to a change τ_y or m or both) will have impact on the distribution of disposable income ($\widetilde{\sigma}^2_{t,y}$). As shown in (8c), except for consumption tax, policy has influence on individual disposable income, which in turn has influence on the dynamics of inequality. We see from (27) that such policies do not have effect on $\sigma^2_{t,y}$, which is rather determined by φ.[5]

The second is an indirect *substitution effect* via changing the composition of the public-private inputs in the production process. A change in the fiscal structure changes the public-private capital ratio (φ) that influences the individual pre-tax and -transfer income, and determines the distribution of the next period investment and income. An increase in the tax structure or a decline in transfer increases φ, and vice versa. The winner and losers from this change are determined by the elasticity of substitution between the public and private capital. As households use the public input to substitute and complement their private investment, if the elasticity of substitution is greater than unity, household with relatively lower private endowment could benefit more than proportional from factor substitution, as this relax credit constraints that impede their investment. The availability of more complementary public investment rather disproportionately benefits the rich, who have a relatively lower marginal product due to diminishing returns to investment. The rich own much of the private resource in the economy.

7. Redistributive policies and inequality dynamics

Both the government's expenditure and financing policies could affect inequality. The government could redistribute income through (i) its investment on public capital, (ii) income transfer, through their financing of, using (iii) consumption and/or (iv) income taxes. Although these are not necessarily mutually exclusive, it is important to examine each individually, since the mechanism through which one has effects on inequality could be independent of the other.

Total differentiation of (21) reveals all channels through which policy could affect the dynamics of inequality:

[5] As long as $m \neq 0 \Leftrightarrow s = 1$, the post- and pre-tax income distribution are different.

$$d\sigma^2_{t+1,k} = \frac{e^{\sigma^2_{t,y}} - 1}{e^{\widetilde{\sigma}^2_{t,y}}} ds^2 + s^2 \frac{e^{\sigma^2_{t,y}}}{e^{\widetilde{\sigma}^2_{t,y}}} d\sigma^2_{t,y} \tag{29}$$

Therefore, the change in inequality in the economy is a result of both changes in before- and after-tax and -transfer income distribution, which are captured in the second and first terms, respectively. Recall that s^2 only relates to $\widetilde{\sigma}^2_{t,y}$ but $\sigma^2_{t,y}$.[6]

7.1. Redistribution through public investment

The substitution effect is given by the change in inequality due to a change in before-tax and -transfer income distribution, which, in turn, is due to a change in φ:

$$\frac{d}{d\varphi} \widetilde{\sigma}^2_{t,y} = s^2 \frac{e^{\sigma^2_{t,y}}}{e^{\widetilde{\sigma}^2_{t,y}}} \frac{d\sigma^2_{t,y}}{d\varphi} \approx \frac{d\sigma^2_{t,y}}{d\varphi} \tag{30}$$

In this case, $ds^2/d\varphi = 0$, which is, particularly, the case if the change in the public-private is due to the change in consumption tax or a change in household preference, β.[7]

Since, from (22) and (30),

$$\frac{d\widetilde{\sigma}^2_{t,y}}{d\varphi} \approx -\frac{2\rho^{-1}}{\varphi} \frac{1}{x_t + 1} \gtrless 0 \text{ if } \rho \lesseqgtr 0 \tag{31}$$

the effects of φ on inequality depends mainly on the *sign* of ρ, which determines the elasticity of substitution between public and private capital.[8] An increase in the public-private capital ratio, independent of the financing method, aggravates (mitigates) inequality dynamics if the elasticity of substitution is lesser (greater) than unity. If the elasticity of substitution is equal to unity, a change in the capital ratio has no effect on inequality dynamics. Intuitively, the degree of substitutability relates to lower inequality persistence, as it affects individual resource constraints that arises due to the imperfection in the credit market.

[6] We see below, $s^2 \approx \widetilde{\sigma}^2_{t,y}/\sigma^2_{t,y}$.

[7] We apply the approximation $e^{\sigma^2_{t,y}} \approx e^{\sigma^2_{t,y}} - 1$, for small $\sigma^2_{t,y}$, in (20).

[8] In deriving (31), we use the approximation $e^{\rho^2(\sigma^2_{t,y} - v^2)} - 1 \approx e^{\rho^2(\sigma^2_{t,y} - v^2)}$.

7.2. Redistribution through transfer

The second channel through which policy affects inequality is through a direct effect on the *ex post* income distribution. First, note that considering (21),

$$s^2 \approx \frac{\widetilde{\sigma}_{t,y}^2}{\sigma_{t,y}^2} = \frac{\overline{y}_t}{\widetilde{y}_t} \tag{32}$$

where s^2 and $1/s^2$ are the ratios of before- and after-tax and -transfer aggregate income and distribution, respectively. Given that s is less than unity, eq. (32) defies the "leaky bucket" redistribution of income. There exists inequality-efficiency trade-offs. Any gains in equity from a decrease in s, say due to an increase in τ_y or m, necessarily follows with an increase in efficiency.

Second, the income effect is given, solely, by the effect of s on $\widetilde{\sigma}_{t,y}^2$, which is derived from (29) as,

$$\frac{\partial \widetilde{\sigma}_{t,y}^2}{\partial s^2} = \frac{e^{\sigma_{t,y}^2} - 1}{e^{\widetilde{\sigma}_{t,y}^2}} \approx \frac{1}{s^2},\ m \neq 0 \tag{33}$$

where the second term is zero, and $\partial \sigma_{t,y}^2 / \partial \varphi = 0$. This could happen ($\varphi$ remains constant), if an increase (a decrease) in income tax is accompanied with a decrease (an increase) in consumption tax in the following proportion,

$$\frac{1-\beta}{(1+\tau_c)^2} d\tau_c = -\frac{1}{(1-\tau_y+m)^2} d\tau_y \tag{34}$$

or, an increase or decrease in the transfer rate should be accompanied with a similar change in consumption tax according to

$$\frac{1-\beta}{(1+\tau_c)^2} d\tau_c = \frac{1}{(1-\tau_y+m)^2} dm \tag{35}$$

Also, s also could change (while φ remains constant) without a change in consumption tax but with a one-to-one change in τ_y and m:

$$\frac{dm}{d\tau_y} = -1 \tag{36}$$

From (31) and (33), the distributional effects of transfer is given by, respectively,

$$\frac{\partial\widetilde{\sigma}_{t,y}^2}{\partial m} \approx \frac{2}{(1-\tau_y+m)^2}\left(-\frac{1}{s}(1-\tau_y)+\frac{1}{\rho\varphi\beta}\frac{1}{x_t+1}\right) \tag{37}$$

The income effect, the first term in (37), is always negative whereas the sign of the second term, the substitution effect, depends on the sign of ρ. It is negative if $\rho < 0$, and conversely. Therefore, $\rho < 0$ implies (37) is negative and redistribution of income leads to a lower inequality. However, this is not necessarily the case if $\rho > 0$. Because, although income redistribution helps to mitigate inequality through its effect on the after-tax and -transfer income distribution it also puts pressure on the resource available for (substitutable) public investment. If $\rho > 0$, from (31) and (37), income transfer leads to a lower inequality if the substitution effect is sufficiently low:

$$\frac{d\widetilde{\sigma}_{t,y}^2}{d\varphi} < 2\beta(1-\tau_y+m) \tag{38}$$

We summarize the above discussion with the following propositions:

Proposition 2. *(i) If the elasticity of substitution between public and private capital is greater than unity, then the greater is the public-private capital ratio, the lower is the persistence of inequality, and conversely. (ii) If the elasticity of substitution is lesser than unity, then income transfer reduces inequality. Otherwise, the effect is non-monotonous. (iii) Efficiency-inequality trade-offs rules the relations of before- and after-tax and -transfer income and distribution.*

7.3. Taxes and inequality persistence

From (31) and (33), the distributional effects of consumption (τ_c) and income (τ_y) taxes are given by, respectively,

$$\frac{\partial\widetilde{\sigma}_{t,y}^2}{\partial\tau_y} = \frac{2}{(1-\tau_y+m)^2}\left[-m\frac{1}{s}-\frac{1}{\beta\rho\varphi}\frac{1}{x_t+1}\right] \tag{39}$$

and

$$\frac{\partial\widetilde{\sigma}_{t,y}^2}{\partial\tau_c} = -\left(\frac{2}{\rho\varphi}\frac{1}{x_t+1}\right)\frac{1}{(1+\tau_c)^2}\frac{1-\beta}{\beta} \gtrless 0 \text{ if } \rho \lessgtr 0 \tag{40}$$

We see from (39) and (40), consumption (τ_c) and income (τ_y) taxes affect inequality dynamics via their impact on φ, which is closely tied with the factor elasticity substitution. The effect of τ_c solely depends on the sign of ρ. However, τ_y has an additional impact on inequality through the income effect. If $\rho > 0$, an increase in τ_y leads to a lower inequality. But, this is not necessarily the case if $\rho < 0$. In the latter case, inequality declines (following an increase in τ_y) if the income effects dominates the substitution effects or,

$$\frac{d\widetilde{\sigma}^2_{t,y}}{d\varphi} < 2\beta m s^{-1}$$

Proposition 3. *(i) If $\rho \geqslant 0$ then a higher income tax (τ_y) decreases inequality persistence. If $\rho < 0$, then the distributional effect depends on the balance between the substituting and income effects. (ii) If $\rho > 0$ ($\rho < 0$), a higher consumption tax (τ_c) decreases (increases) inequality persistence.*

7.4. Policy ranking

Ranking of various policies can be done based on their relative impacts on the distributional dynamics. Since the effects of different policies are closely related to the factor elasticity of substitution, any comparison shall consider the values of the elasticity of substitution between public and private investment. For the reason mentioned above, consumption tax (τ_c) is expected to have the least impact in the economy in terms of affecting the dynamics of the distribution of income. In addition, the substitution effect of τ_y is much stronger than that of τ_c due to its distortionary effect on private investment. This is evident by:

$$\varphi'_{\tau_y} > \varphi'_{\tau_c} \tag{41}$$

A higher τ_y not only increases g but also decreases k whereas τ_c has no such distortionary impact on the economy.

Although τ_y seems to be superior than τ_c in terms of its policy effects, under certain conditions τ_c may be a preferable policy. For instance, in the case $\rho < 0$, whether slashing the income tax may lead to a lower inequality or not hinges on the balance between the income and substitution effects. A similar reduction in consumption will lead to a definite lower inequality, however. Comparing and contrasting (39) and (40), if the substitution effects is too small, then consumption tax is a clearly preferable policy.

Proposition 4. *(i) When $\rho \geqslant 0$, then τ_y is superior to τ_c. When $\rho < 0$, the sufficient condition for τ_c to dominate τ_y is given by (). (ii) If $\rho < 0$, m is superior to φ. When $\rho > 0$,*

Proposition 5. *:*

$$\frac{d\widetilde{\sigma}^2_{t,y}}{d\varphi} < 2\beta m s^{-1}$$

8. Policy, inequality, and growth

8.1. Short-run effects

From (14), output growth lags one period from capital growth, in the aggregate: $\gamma_t^y = \gamma_{t+1}^k$. Thus, the growth rate of per capita output growth at time t is given from (17) and (19):

$$\gamma_t^y = \ln\left(\psi\alpha^{\frac{1}{\rho}}\left(x_t+1\right)^{\frac{1}{\rho}}\right) + \frac{1-\rho}{2}\ln\frac{e^{\sigma^2_{t+1,k}}+s^2-1}{s^2} \tag{42}$$

Policy affects growth through two channels. The first is through the traditional Barro channel. Although public investment could promote efficiency through a positive complementary and productivity effects on private capital, its financing could distort private saving and investment leading to a negative impact. Such effects is captured in the first term of (42), in ψ and x_t; however, in contrast to Barro, x_t is also a function of inequality here. The second is through its effect on income inequality. This is captured in the second term of (42).

What is striking is only the substitution effects of inequality is important to short run growth. This is, in particular, evident when substituting (21) into (42),

$$\gamma_t^y = \ln\left(\psi\alpha^{\frac{1}{\rho}}\left(x_t+1\right)^{\frac{1}{\rho}}\right) + \frac{1-\rho}{2}\sigma^2_{t,y} \tag{43}$$

Thus, only before-tax and -transfer inequality is important for short-run growth.[9] s^2 has no role in the short-run growth of the economy, which implies whatever effect policy has on after-tax and -transfer income distribution is not relevant for growth.[10]

Therefore, together with (32) and (43), we have the following proposition:

[9] Ostry et al. (2014) identify before- and after-tax and -transfer income distributions as "market" and "net inequality", respectively.

[10] Note also that we will arrive to (43) if we start $m = 0 \Leftrightarrow s = 1$.

Proposition 6. *(i) The effects of taxes and transfers on after-tax and -transfer income distribution have no growth but level effects in the short-run. (ii) Taxes and transfer increases growth to the extent they increase before-tax and -transfer income distribution.*

8.2. Steady state effects

Steady state of growth is given by, from (43) and (24),

$$\gamma^y = \ln\left(\psi\alpha^{\frac{1}{\rho}}\left(x_t+1\right)^{\frac{1}{\rho}}\right) + \frac{1-\rho}{2}\sigma_y^2 \tag{44}$$

where

$$\sigma_y^2 = \rho^{-2}\ln\left(\frac{x^2}{\left(x+1\right)^2}\left(\left(s^2\left(e^{\sigma_y^2}-1\right)+1\right)^{\rho^2}-1\right)+1\right)+\upsilon^2 \tag{45}$$

and x is define in (24c). We see from (44) and (45) that s plays a role in the steady state growth.

8.3. Growth and welfare maximization

9. Closed form solution

By applying L'hopital's rule in (21) and (22), we can derive the distribution of capital and income, respectively, in the Cobb-Douglas case ($\rho = 0$),

$$\sigma_{t+1,k}^2 = \ln\left(s^2\left(e^{(1-\alpha)^2\sigma_{t,,k}^2+\upsilon^2}-1\right)+1\right) \tag{46}$$

and

$$\sigma_{t,y}^2 = \left(1-\alpha\right)^2\sigma_{t,,k}^2+\upsilon^2 \tag{47}$$

where $1-\alpha$ is the factor share of k_{it} in this case. We see from (46), the substitution effects of inequality is disappeared and there is only one channel that policy could affect inequality, which is through the income effects, or s^2. In the Cobb-Douglas case, factor shares are constant, and hence independent of the factor proportion leading to distribution neutrality of policy.

Aggregate income in the Cobb-Douglas case is given by,

$$y_t = \varphi^\alpha k_t e^{0.5\sigma_{t,k}^2\alpha(\alpha-1)} \tag{48}$$

from which we easily derive the growth rate per capita output,

$$\gamma_t^y = (1 - \tau_y + m)\,\varphi^\alpha - 0.5\sigma_{t,k}^2\alpha\,(1-\alpha)$$
$$= (1 - \tau_y + m)\,\varphi^\alpha - \frac{1}{2}\alpha\frac{\sigma_{t,y}^2 - \upsilon^2}{1-\alpha} \tag{49}$$

(49) is consistent with (43) that *only* the before-tax and -transfer distribution is important for short-run growth, which is independent of policy choices (47).

The steady state inequality and growth are given by, respectively,

$$e^{\sigma_k^2} - 1 = s^2\left(e^{\upsilon^2 + (1-\alpha)^2\sigma_k^2} - 1\right) \tag{50}$$

and,

$$\gamma^y = (1 - \tau_y + m)\,\varphi^\alpha + 0.5\sigma_k^2\alpha\,(\alpha - 1) \tag{51}$$

where σ_k^2 is implicitly defined in (50). Revoking our approximation for (49), these become, respectively,

$$\sigma_k^2 \approx s^2\upsilon^2/\left(1 - (1-\alpha)^2\right)$$

and

$$\gamma^y \approx (1 - \tau_y + m)\,\varphi^\alpha - \frac{1}{2}\alpha\,(1-\alpha)\,\frac{s^2\upsilon^2}{1-(1-\alpha)^2} \tag{52}$$

From (52), the ratio of before and after tax and transfer inequality is proportional to the steady state growth.

The following proposition summarizes the above discussion:

Proposition 7. *(i) In the steady state, both before and after-tax and -transfer income distributions are important to growth. (ii) In the Cobb-Douglas case, only the effects of taxes and transfers on after-tax and -transfer income distribution is relevant to growth. (iii) Taxes and transfer increase steady-state growth to the extent they increase before and after-tax and -transfer income distribution.*

10. Conclusion

The paper has studied the mechanism through which various fiscal policies have effect on inequality-growth trade-offs, in a growth model where both growth and inequality are endogenously determined. Two main channels have been identified through which fiscal policy could have influence on the inequality-growth nexus: a direct income effect on the post-tax and -transfer income distribution and an indirect substitution effect on the private-public capital ratio that influences the pre-tax and -transfer income distribution. We found that, with lump sum transfer, the distribution of investment is similar to that of consumption distribution and the distribution of post-tax and -transfer income but it is greater than the pre-tax and -transfer income distribution. Ranking various policies based on their relative impacts on the distributional dynamics, consumption tax has the least impact on inequality, as it merely influence inequality through the substitution effect. In contrast, income tax and transfer have a relatively stronger impact on inequality through both the substitution and income effects. We also found that the relationship between before-tax and -transfer inequality and after-tax and -transfer inequality, market and net inequality, and growth could be different. In the short-run, only market inequality is important for growth whereas both are important in the long run.

A. Aggregation and distribution

Aggregation is simplified due to the assumption of a lognormal distribution of wealth.

We use the expectation and variance notation E and V to denote aggregation and variance of a variable, respectively. Aggregate capital is thus defined as:

$$k_t \equiv \mathrm{E}\, k_{it} = \int_i k_{it}$$

The distribution of wealth (k_{it}) is represented by its log variance,

$$\sigma^2_{t,k} = \mathrm{V} \ln k_{it}$$

where $\sigma^2_{t,k}$ is the log-linear distribution of wealth.

A.1. Aggregate income and variance

Suppose x_i is a log-normal random variable, then $x_i + 1$ is a *shifted* log-normal where $x_i + 1 > 0$:

$$\ln x_i \sim N\left(\mu_1, \sigma_1^2\right) \text{ and } \ln\left(x_i + 1\right) \sim N\left(\mu_2, \sigma_2^2\right). \tag{A.1}$$

The mean and the variance of x_i are given by the following relations:

$$\mathrm{E}\, x_i = e^{\mu_2 + 0.5\sigma_2^2} - 1 = e^{\mu_1 + 0.5\sigma_1^2} \tag{A.2}$$

$$\mathrm{V}\, x_i = e^{2\mu_2 + \sigma_2^2}\left(e^{\sigma_2^2} - 1\right) = e^{2\mu_1 + \sigma_1^2}\left(e^{\sigma_1^2} - 1\right) \tag{A.3}$$

From (A.2) and (A.3), one may solve for σ_2^2 and μ_2 in terms of x and σ_1^2 to obtain,

$$\sigma_2^2 = \ln\left(x^2\left(e^{\sigma_1^2} - 1\right) / \left(x + 1\right)^2 + 1\right) \tag{A.4}$$

$$\mu_2 = \ln\left[\left(x + 1\right)\left(\frac{x^2}{\left(x + 1\right)^2}\left(e^{\sigma_1^2} - 1\right) + 1\right)^{-0.5}\right] \tag{A.5}$$

where $z \equiv \mathrm{E}\, x_i$.

Then, to aggregating y_{it} first, rewrite (3),

$$\ln y_{it} = \ln \epsilon_{it} g_t \alpha^{\frac{1}{\rho}} + \rho^{-1} \ln\left(1 + x_{it}\right) \tag{A.6}$$

where

$$x_t^i \equiv \left(1 - \alpha\right)\alpha^{-1}\left(k_{it}/g_t\right)^{\rho} \text{ and } \mathrm{E}\, x_{it} \equiv x_t \tag{A.7}$$

Note that x_{it} is log-normal, as k_{it} is log-normal.

Then, compute the mean and variance of (A.6), using (A.4) and (A.5), respectively,

$$\begin{aligned}\mathrm{E}\ln y_{it} &= \ln g_t \alpha^{1/\rho} - 0.5 v^2 + \rho^{-1}\,\mathrm{E}\left[\ln\left(1 + x_t^i\right)\right] \\ &= \ln g_t \alpha^{1/\rho} - 0.5 v^2 + \rho^{-1} \ln\left(\left(x_t + 1\right)\left(\frac{x_t^2}{\left(x_t + 1\right)^2}\left(e^{\rho^2 \sigma_{t,k}^2} - 1\right) + 1\right)^{-0.5}\right)\end{aligned} \tag{A.8}$$

and

$$\begin{aligned}\sigma_{t,y}^2 \equiv \mathrm{V}\left[\ln y_{it}\right] &= \rho^{-2}\,\mathrm{V}\ln\left(1+x_{it}\right)+\upsilon^2 \\ &= \rho^{-2}\ln\left(\frac{x_t^2}{\left(x_t+1\right)^2}\left(e^{\rho^2\sigma_{t,k}^2}-1\right)+1\right)+\upsilon^2 \end{aligned} \tag{A.9}$$

where

$$x_t \equiv \left(1-\alpha\right)\alpha^{-1}\varphi^{-\rho}e^{\sigma_{t,k}^2 0.5\rho(\rho-1)}$$

Since $\ln y_t = \mathrm{E}\left[\ln y_{it}\right] + 0.5\,\mathrm{var}\left[\ln y_{it}\right]$, the aggregate income (y_t) is given by

$$y_t = g_t\alpha^{\frac{1}{\rho}}\left(x_t+1\right)^{\frac{1}{\rho}}\left(x_t^2\left(e^{\rho^2\sigma_{t,k}^2}-1\right)\left(x_t+1\right)^{-2}+1\right)^{(1-\rho)/\left(2\rho^2\right)}$$

A.2. Inequality dynamics

From (8b) and (8a), we have,

$$c_{it}\left(1+\tau_c\right) = \left(\left(1-\tau_y\right)y_{it}+my_t\right)\left(1-\beta\right) \tag{A.10}$$
$$k_{it+1} = \left(\left(1-\tau_y\right)y_{it}+my_t\right)\beta \tag{A.11}$$

which lead to the following relation,

$$\mathrm{V}\ln k_{it+1} = \mathrm{V}\ln c_{it} = \mathrm{V}\left[\ln\left(\left(1-\tau_y\right)y_{it}+y_t m\right)\right] \tag{A.12}$$

That is the distribution of current consumption, post tax income and next period investment are equal. The last term is then solved following similar procedures that are discussed above:

$$\sigma_{t+1,k}^2 = \sigma_{t,c}^2 = \ln\left(s^2\left(e^{\sigma_{t,y}^2}-1\right)+1\right) \tag{A.13}$$

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