# Optimal Fiscal Policy in a dollarised economy

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**Abstract**

This paper uses a New Keynesian Dynamic Stochastic General Equilibrium, (DSGE) model developed for dollarised economy to investigate how fiscal policy should be conducted in Zimbabwe.We assume fiscal policy is transmitted through the sovereign risk premium channel.We conduct experiments with simple and optimal simple fiscal policy rules using low and high sovereign risk premium parameters.Our results find that optimal fiscal policy should be procyclical.

JEL Codes: E32, E62, E42

**Keywords:** Pro-cyclical Fiscal Policy, New Keynesian DSGE, Sovereign risk premium, Dollarisation, Zimbabwe

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# 1 Introduction

Zimbababwe dollarised February 2009 after experiencing the second biggest hyperinflation in history. Dollarisation in Zimbabwe led to the abandonment of the local currency and the subsequent loss of monetary policy tools for controlling the economy. This also increased the role of fiscal policy as stabilisattion tool.

This paper uses a standard New keynesian DSGE model developed for a dollarised economy to investigate how fiscal policy should be conducted to stabilise a dollarised economy.The modelling framework is similar to the class of Gali and Monaceli (2008).

We incorporate additional features. First, we assume that nominal interest rates and US dollar supply follow an exogenous process. This is a major difference from standard DSGE settings. Second, using arguments by Corsetti, (2013), we assume the sovereign risk channel of fiscal policy is present in Zimbabwe and that optimal fiscal policy is procyclical. Third, we define soverign risk premium as an increasing function of government bond holdings. Fourth, we let increases (decreases) in Government bond holdings increase (decrease) private sector borrowing costs through changes in the sovereign risk premium. The risk premium for developing countries is substantially high compared to emerging and developed countries. Even though Zimbabwe uses the US dollar, interest rates of about 10 percent per annum on US dollars borrowings are normal. It can be assumed that norminal interests rates in Zimbabwe should compare with zero bound interest rates in the United States of America but this is not the case. Although the exact relationship between the sovereign risk premium and interest rates is unknown in Zimbabwe, the difference in the Federal Funds Rates and prevailing interest rates in Zimbabwe supports our assumption that part of the risk premium is attributable to the sovereign risk premium. We believe that our assumption that the transmission of fiscal policy through the risk premium should not draw controversies. Fifth, fiscal policy actions for government are guided by a fiscal policy rule suggested by Kirsanova, (2007). Sixth, we assume money plays a role in DSGE models see Seitz and Schmidt, (2014) and we include money-in-the production function proposals by Canova and Menz, (2011) and Benchimol, (2014). Our key findings are: First, optimal fiscal policy is procyclical. Second, foreign interest rate shock should be accomodated by fiscal contraction Third, a foreign inflation shock is expansionary.

In general there are few emperical studies that include fiscal policy in Dynamic Stochastic General Equilibrium, (DSGE) models as compared to models with monetary policy Leeper, (2010).

As a consequence there is raging debate on the effects fiscal policy DSGE models, Eggertsoon, (2010).

Turning to studies on fiscal policy for developing countries. Gavin and Perroti (1997) were the first to point out that fiscal policy for Latin American countries is procyclical and Talvi and Vegh, (2005) found that procyclical ficsal policy is to a large extent adeveloping world phenomenon. Emperical research on fiscal policy specific to dollarised countries is scarce. The Peruvian economy is, however, is a fairly well researched case study for highly dollarised economies see Callisto, Montoro and Tuesta, (2013). Palacios-Salguero, (2010) investigates optimal fiscal policy in Peru and finds that optimal fiscal policy should should take into account the deviations from the steady state of the amount of government spending, debt.and inflation. According to the best of our knowledge there are no studies on optimal fiscal policy in fully dollarised economies such as Zimbabwe that use New Keynesian DSGE models.

The rest of the paper is organized as follows. Section 2 presents the model. Section 3 discuses the equilibrium. Section 4 discuses the simulation results. Section 5 concludes.

## 2 The Model

We follow a tracable small open economy modelling framework presented by Woodford (2005), Clarida, Gali, and Gertler (1999) and Gali and Monaceli (2005, 2008) among others and incorporate fiscal policy. We let foreign interest rate, money supply, foreingn inflation and domestic economy productivity to follow idiosyncratic shock process. We use a fiscal rule similar to Kirsanova et al, (2007). We assume sovereign risk premium transmission channel of fiscal policy with the sovereign risk premium functional form proposed by Schmitt-Grohe and Uribe (2003). Our framework allows for monopolistic competition and calvo price setting in the goods market.

The economy inhabited by households, firms and government. The rest of the world sets interest rates and supplies US dollars. Households receive US dollars from the rest of the world and invest firms and government bonds Households charge a risk premium over and above the foreign interest rate for holding government bonds. Further, households supply labour, receive wages and pay taxes .Firms produce goods by using US dollars and household labour and constant return technologies. The goods produced by firms are sold to households, government and the rest of the world economy. Government actions are guided by fiscal rules.Government expenditures are financed by taxes and bonds. We assume Government can default on its obligations

### 2.1 Households

The representative households are infinetly lived. Households supply labour and US dollars to firms. Households are constrained by their budget but seek to maximise the following expected lifetime utility:

$$E_t \sum_{t=0}^{\infty} \beta U(C_t, N_t, G_t, Ms\$_t) \tag{1}$$

The utility function $U(C_t, N_t, G_t)$ given by

$$U(C_t, N_t, G_t) = (1-\chi)\log C_t + \chi \log(G_t) - \frac{{N_t}^{1-\varphi}}{1-\varphi} + \frac{M\$_t}{P_t} \tag{2}$$

where $E_t$ is conditional on time t, $\beta \in [0,1]$ is the discount factor, $G_t$ is an index for Government consumption, $N_t$ is labour supply normalised to 1 $0 \leq N_t \leq 1$ , $\chi$ is relative elastisticity of government consumption relative to consumption in the private sector, $Ms\$_t$ represents US dollars holdings and $\varphi$ is marginal elasticity of labour supply.

The households seek to maximise their utility subject to the following budget constraint:

$$P_t C_t + Ms\$_t + E_t \left\{ \left[(1+i_t^*)\Psi(B_t)\right]^{-1} B_t \right\} = B_t + W_t N_t + Ms\$_{t-1} - T \tag{3}$$

where $P_t$ is the represents aggregate price of consumed goods in the domestic economy, $\Psi(B_t)$ is the risk premium on government bonds, $B_t$ denotes bond holdings at time period t, $W_t$ represents wages, $N_t$ is labour, $\Pi_t$ donotes firm profits, $Ms\$_t$ denotes US dollar holdings, and $T_t$ denotes lumpsum government tax collections. Households earn income from wages, profits from firms and a return, $(1+r_t)\Psi(B_t)$, from holding government bonds, which they charge a risk premium in addition to the norminal interest rate.

#### 2.1.1 Risk Premium

The sovereign risk premium,$\Psi(B_t)$ is a an increasing function of government debt, grosss domestic product and is represented as follows

$$\Psi(B_t) = \delta B_t \tag{4}$$

The intuition is that households charge a premium as an insurence against unforeseen losses in government bonds else households would prefer save their money in their bank accounts or at a secure places in their homes.

First order conditions for the small open economy household problem are given by:

$$\frac{W_t}{P_{d,t}} = \frac{C_t N_t^{\varphi}}{1-\chi} \tag{5}$$

The household labour supply decisions are influenced by lumpsum government taxes and preferences for government produced goods.

$$\beta(1+i_t^*)\Psi(B_t)E_t\left\{\frac{C_t}{C_{t+1}}\frac{P_{d,t}}{P_{d,t+1}}\right\} = 1 \tag{6}$$

where $R_t$ represents the an exogenous interest rate plus a default risk premium on government bonds.

The marginal utility of holding foreign currency balances for households is given by the following equation

$$\frac{\frac{M_t}{P_t}}{C_t} = \frac{R_t}{1+R_t} \tag{7}$$

log linearising we get

$$w_t = c_t + \varphi n_t - \log(1-\chi) \tag{8}$$

Where equation (8), represents the the labour supply equation.

$$c_t = E_t c_{t+1} - (i_t^* + \delta\widehat{b}_t - E_t\pi_{t+1} - \rho) \tag{9}$$

where $\delta\widehat{b}_t$ is the bond risk premium and $\delta$ is the coefficient of government bond stock elasticity, $\rho = -\log\beta$ is equilibrium real interest rate and $i_t^*$ is the norminal interest rate st by the rest of the world economy.

$$md\$_t = p_t + c_t - \eta(i_t^* + \delta\widehat{b}_t) \tag{10}$$

where, $md\$_t$ represents the demand for US dollars in the economy. Intutively equation (10) implies that the relationship between money and inflation holds in equilibrium if consumption and interest rates are independent of US dollar holdings. In order to solve the model, US dollar demand should equal to US dollar supply. The US dollars are supplied by the from the rest of the world economy. We represent US dollar supply as a shock process.as shown below

$$ms\$_t = \rho^{ms\$} ms_{t-1} + \varepsilon^{ms\$} \tag{11}$$

## 2.2 The Productive Sectors

We assume there exists a continuum of firms that operate in monopolistic business environment.

### 2.2.1 Firms

Firms produce differenciated goods. The money- in- the production for firm $j$ is a variant of one proposed by Benchimol, (2014) and is given by

$$Y_t(f) = \left( \frac{M\$_t}{P_t}(j) \right)^{\alpha_m} A_t(j) N_t(j) \tag{12}$$

The firms aggregate production function is represented as follows

$$Y_t = \left( \frac{M\$_t}{P_t} \right)^{\alpha_m} A_t N_t \tag{13}$$

where $A_t$ denotes country wide firm productivity parameter, $Y_t$ is firm output, $N_t$ represents labour force,$A_t$ is technology, $M\$_t$ is the money demand gap,and $\alpha_m$ is the coeeficient of the money demand gap.

Expressing the equation in gap form and log linearising we obtain

$$\widehat{y}_t = \widehat{n}_t + \alpha_m \widehat{m\$} + a_t \tag{14}$$

where $\alpha_m$ is the coeeficient of the money demand gap

**Calvo pricing** We add the price stickiness concept popularised by calvo (1983) and denote $\omega$ to represent the probability of a firm not changing prices at a given time period, we set $1 - \omega$ to represent the fraction of firms that can optimally change prices at a given time perid. Firms allowed to change prices set their optimal prices to prices to $\overline{p}_t$. The optimised price selected by the firms allowed to change prices at time $t$ seek to maximise the following expected profit function:

$$E_t\left\{\sum_{x=0}^{\infty}\omega^x R_{t,t+x}Y_{t+x}(\overline{P}_t-\frac{\rho}{1-\rho}P_{t+x}MC_{t+x})\right\}=0 \tag{15}$$

where, $\varepsilon/(\varepsilon-1)$ denotes the inovation to price markup, $MC_t^n$ represents price markup, and $R_{t,t+x}$ represents the return of a risk free security.

Using the Dixit-Stiglitz (1977) aggregator, we represent firm aggregate price index as shown in equation 9:

$$p_t=((1-\omega)(\overline{p})^{1-\epsilon}+\omega(p_t)^{1-\epsilon})^{\frac{1}{1-\epsilon}} \tag{16}$$

Using the dictacts of law of motion we represent $p$ as follows

$$p_t=(1-\omega(\overline{p}_t)+\omega(p_{t-1}) \tag{17}$$

We write the optimal price chosen my firms that are allowed to change prices as:

$$\overline{p}_t=\mu+(1-\beta\omega)\sum_{x=0}^{\infty}(\beta\omega)^x E_t(mc_{t+x}+p_t) \tag{18}$$

The intution is that firms that are allowed to change prices would choose a adjust prices would set prices after considering current prices and marginal cost as well as future marginal costs. Inflation would increase if the firms's steady state markups are below the anticipated average markups. Precisely, firms would want to set prices that are above average to maximise their profits.

Domestic inflation will therefore be given by sum of expected future inflation and expected real marginal costs as shown below;

$$\pi_{d,t}=\beta E_t\{\pi_{t+1}\}+\lambda\widehat{mc}_t \tag{19}$$

where, $\lambda=\frac{(1-\beta\omega)(1-\omega)}{\omega}$ and $\widehat{mc}_t$ is the real marginal cost for firms Real marginal cost is represented by:

$$\widehat{mc}_t=\frac{(1+(\varphi-1)(1-\chi))}{(1-\chi)}\widehat{y}_t-\frac{\chi}{(1-\chi)}\widehat{g}_t-(2\alpha_m-1)\widehat{m\$}_t \tag{20}$$

### 2.2.2 New Keynesian Philips curve

Using the standard representation for the standard New Keynesian Philips curve and a representation for firms marginal costs we get a representation for domestic inflation shown below:

$$\pi_{d,t} = \beta E_t \pi_{d,t+1} + \lambda \frac{(1+(\varphi-1)(1-\chi))}{(1-\chi)} \widehat{y}_t - \frac{\lambda\chi}{(1-\chi)} \widehat{g}_t - \lambda(2\alpha_m - 1)\widehat{m\$}_t \tag{21}$$

Real marginal cost is negatively related to government spending and US dollar demand gap. For a give output level, government spending crowds out domestic consumption. The other explanation is that an increase in government spending causes an appreciation and reduces the real wage..

### 2.2.3 Market Clearing Condition

Market clearing implies that all the produced products are consumed. The equilibrium condition is given by:

$$y_t = \chi g_t + (1-\chi) c_t$$

$$c_t = \frac{1}{(1-\chi)} y_t - \frac{\chi}{(1-\chi)} g_t \tag{22}$$

### 2.2.4 Dynamic IS curve

We derive the Dynamic IS curve. Using (9) and (22) see also Eggertson, (2010) we derive the standard new keynesian IS curve

$$\frac{1}{(1-\chi)} y_t - \frac{\chi}{(1-\chi)} g_t = \frac{1}{(1-\chi)} E_t y_{t+1} - \frac{\chi}{(1-\chi)} E_t g_{t+1} - (i_t^* + \delta \widehat{b}_t + E_t \pi_{t+1} - \rho)$$

We rearrage by multiplying throughout by $(1-\chi)$ and make output, $y_t$ the subject of the formula as shown below

$$y_t - \chi g_t = E_t y_{t+1} - (1-\chi)(i_t^* + \delta \widehat{b}_t + E_t \pi_{t+1} - \rho) - \chi E_t g_{t+1}$$

We make output, $y_t$ the subject of the formula as shown below:

$$y_t = E_t y_{t+1} - (1-\chi)(i_t^* + \delta\widehat{b}_t + E_t\pi_{t+1} - \rho) + \chi g_t - \chi E_t g_{t+1}$$

Simplifying by factoring out $\chi$ we get:

$$y_t = E_t y_{t+1} - (1-\chi)(i_t^* + \delta\widehat{b}_t + E_t\pi_{t+1} - \rho) - \chi(E_t g_{t+1} - g_t)$$

Using identity $E_t \Delta g_{t+1} = (E_t g_{t+1} - g_t)$ obtain present output by the following expression:

$$y_t = E_t y_{t+1} - (1-\chi)(i_t^* + \delta\widehat{b}_t + E_t\pi_{t+1} - \rho) - \chi E_t \Delta g_{t+1} \tag{23}$$

We represent real interest rate by, $r_t$:

$$r_t = i_t^* + \delta\widehat{b}_t - E_t\pi_{t+1} \tag{24}$$

Inserting (24) in (23) we represent ouput as follows:

$$y_t = E_t y_{t+1} - (1-\chi)(r_t - \rho) - \chi E_t \Delta g_{t+1} \tag{25}$$

Using the same steps we represent natural output, $y_t^n$, in terms of natural government spending, $g_t^n$, and natural interest rate interest rate, $r_t^n$, as shown below:

$$y_t^n = E_t y_{t+1}^n - (1-\chi)(r_t^n - \rho) - \chi E_t \Delta g_{t+1}^n \tag{26}$$

We define output gap,$\widehat{y}_t$,as follows:

$$\widehat{y}_t = y_t - y_t^n \tag{27}$$

Subtracting (26) from (25) we represent the dynamic IS in terms of the variables in their steady state

$$\widehat{y}_t = (E_t y_{t+1} - E_t y_{t+1}^n) - (1-\chi)(r_t - \rho + \rho - r_t^n) - \chi(E_t \Delta g_{t+1} - E_t \Delta g_{t+1}^n) \tag{28}$$

We eliminate the equilibrium interest rate and obtain the following:

$$\widehat{y}_t = (E_t y_{t+1} - E_t y_{t+1}^n) - (1-\chi)(i_t^* + \delta\widehat{b}_t - E_t\pi_{t+1} - r_t^n) - \chi(E_t \Delta g_{t+1} - E_t \Delta g_{t+1}^n) \tag{29}$$

Finally we simply and present the equation for output gap as follows:

$$\widehat{y}_t = E_t\widehat{y}_{t+1} - (1-\chi)(i_t^* + \delta\widehat{b}_t - E_t\pi_{t+1} - r_t^n) - \chi E_t\Delta\widehat{g}_{t+1} \tag{30}$$

We apply the transversality condition $\lim_{T\to\infty} E_t y_{t+T} = \lim_{T\to\infty} E_t g_{t+T} = 0$ and solving (29) we obtain

$$\widehat{y}_t = \chi\widehat{g}_t - (1-\chi)\left(E_t\sum_{k=0}^{\infty}(\widetilde{i}_{t+k}^* + \delta\widehat{b}_{t+1} - \pi_{t+k+1} - r_t^n)\right) \tag{31}$$

Intutively equation (30) tell us that the deviations in the is output gap will emanate from changes in summation of differences in the real interest rate and the natural interest rate consumption tax and deviations in government spending gap with the weights attached being, $\chi$ and $-(1-\chi)$,respectively.

#### 2.2.5 Natural Interest rate

The natural interest rate is represented as follows

$$r_t^n = \rho + \frac{\varphi+1}{(1+\varphi(1-\chi))}E_t\Delta a_{t+1} + \frac{\chi}{(1-\chi)}\frac{(1-(1+\varphi(1-\chi)))}{(1+(\varphi-1)(1-\chi))}E_t\Delta g_{t+1}^n + \frac{(2\alpha_m-1)}{(1+(\varphi-1)(1-\chi))}E_t\Delta m\$_{t+1}^n \tag{32}$$

The deviation of natural interest rate from the equilibrium rate is represented as function of expected natural government spending, technological advances; US dollar demand gap.as shown below:

$$\widehat{r}_t^n = \phi_1 E_t\Delta a_{t+1} + \phi_2 E_t\Delta g_{t+1}^n + \phi_3 E_t\Delta m\$_{t+1}^n \tag{33}$$

where, $\phi_1 = \frac{\varphi+1}{(1+\varphi(1-\chi))}$, $\phi_2 = \frac{\chi}{(1-\chi)}\frac{(1-(1+\varphi(1-\chi)))}{(1+(\varphi-1)(1-\chi))}$, and $\phi_3 = \frac{(2\alpha_m-1)}{(1+(\varphi-1)(1-\chi))}$.

### 2.3 Fiscal Authority and Foreign Monetary Policy

#### 2.3.1 Fiscal Policy

This paper investigates government spending effects in a debt constrained econonomy. We assume for simplicity that government allocate expenditures to domestic goods to produce a public good. We denote the index of consumption for government goods by $G_t$ which is represented by the following expression;

$$G_t \equiv \left( \int_0^1 G_t\left(i\right)^{\frac{\varepsilon-1}{\varepsilon}} df \right)^{\frac{\varepsilon-1}{\varepsilon}} \tag{34}$$

where,

$$G_t\left(f\right) \equiv \left(\frac{P_{d,t}}{P_t}\right)^{-\upsilon} G_t \tag{35}$$

We introduce a countercyclical fiscal rule,that is a function of output, inflation and debt given by the following representation:

$$\widehat{g}_t = -\phi_y \widehat{y}_t - \phi_\pi \pi_t - \phi_d \widehat{b}_t \tag{36}$$

where, $\phi_y$,$\phi_\pi$ and $\phi_d$ are stabilisation parameters.

Government finances its consumption from lumpsum taxes and issuances of short term bonds to finance the government deficit. The loglinear form of the government bond gap is given by:

$$\widehat{b}_t = i_t^* + (1+\delta)\widehat{b}_{t-1} + \widehat{g}_t \tag{37}$$

where, $\widehat{b}_t$, represents the governenmnet bond issuance gap.

#### 2.3.2 Optimal Fiscal Policy

Optimal fiscal policy seeks to maximise the welfare of the economic agents by ensuring that their loss function is minimised. The the social loss function is represented as follows:

$$L_t = \phi_\pi \pi_t^2 + \phi_{\widehat{y}} \widehat{y}_t^2 + \phi_{\widehat{b}} \widehat{b}_t^2 \tag{38}$$

#### 2.3.3 Rest of the world Monetary Policy

The small open economy is fully dollarised, interest rates are an unkown exogenous policy shock i.e $i_t^* = \rho^{i_t^*} a_{t-1} + \nu^{i_t^*}$.

Money supply is exogenous process. The supply of US dollars from the rest of the world economy is given by $ms\$_t = \rho^{ms\$} ms\$_{t-1} + \varepsilon^{ms\$}$.

# 3 The equilibrium

The main Characteristics of the small open economy in log linear form are given by:

1. The IS curve for the domestic economy:

$$\widehat{y}_t = E_t\widehat{y}_{t+1} - (1-\chi)(i_t^* + \delta\widehat{b}_t - E_t\pi_{t+1} - \widehat{r}_t^n) - \chi E_t\Delta\widehat{g}_{t+1}$$

   where, $\widehat{r}_t^n$ denotes the deviation of natural interest rate from the equilibrium rate.

2. The Philips curve for the domestic economy:

$$\pi_{d,t} = \beta E_t\pi_{d,t+1} + \lambda\frac{(1+(\varphi-1)(1-\chi))}{(1-\chi)}\widehat{y}_t - \frac{\lambda\chi}{(1-\chi)}\widehat{g}_t - \lambda(2\alpha_m - 1)\widehat{m\$}_t$$

3. Fiscal rule :

$$\widehat{g}_t = -\phi_y\widehat{y}_t - \phi_\pi\pi_t - \phi_b\widehat{b}_t$$

   where, $\phi_y$, $\phi_\pi$ and $\phi_b$, are stabilisation parameters.

4. Government debt is given by:

$$\widehat{b}_t = i_t^* + (1+\delta)\widehat{b}_{t-1} + \widehat{g}_t$$

   where, $\delta\widehat{b}_{t-1}$, repreesnts the risk premium on government bonds.

5. Domestic consumer price index

$$\pi_t = (1-\alpha)\pi_{d,t} + \alpha p_{f,t}$$

   Where $\pi_t$ is domestic inflation,$\alpha$ is the country's degree of openness and $p_{f,t}$ is foreign inflation.

6. Foreign inflation $p_{f,t}$ is represented as follows:

$$p_{f,t} = \rho^{p_{f,t}} p_{f,t-1} + \varepsilon^{p_{f,t}}$$

   where $\varepsilon^{p_{f,t}}$ is shock to foreign inflation.

7. Representation for output is shown below

$$\widehat{y}_t = \widehat{n}_t + a_t + \alpha_m \widehat{m\$}_t$$

where $n$ and $a$ and represents labour and productivity, respectively.

8. Demand for for US dollar holdings is given by:

$$md\$_t = \pi_{d,t} + \widehat{y}_t - \eta(i_t^* + \delta\widehat{b}_t)$$

9. The deviation of natural interest rate from the equilibrium rate for the small open economy is given by:

$$\widehat{r}_t^n = \phi_1 E_t \Delta a_{t+1} + \phi_2 E_t \Delta g_{t+1}^n + \phi_3 E_t \Delta m\$_{t+1}^n$$

where, $\phi_1 = \frac{\varphi+1}{(1+\varphi(1-\chi))}$, $\phi_2 = \frac{\chi}{(1-\chi)} \frac{(1-(1+\varphi(1-\chi)))}{(1+(\varphi-1)(1-\chi))}$ ,and $\phi_3 = \frac{(2\alpha_m-1)}{(1+(\varphi-1)(1-\chi))}$.

10. Money supply from the rest of the world economy is represented as follows:

$$ms\$_t = \rho^{ms\$} ms\$_{t-1} + \varepsilon^{ms\$}$$

where, $\varepsilon^{ms}$ is a shock to the supply of US dollars from the rest of the world economy.

11. Rest of the world monetary policy (i.e policy rate for the dollarised economy)

$$i_t^* = \rho^{i_t^*} i_{t-1}^* + \varepsilon^{i_t^*}$$

where $\varepsilon^{\widetilde{i^*}}$ is a shock to foreign interest rate.

12. Technology shock process is given as shown below

$$a_t = \rho^a a_{t-1} + \varepsilon^a$$

where, $\varepsilon^a$ is a shock to domestic productivity.

13. Interest rate gap

$$\widehat{r}_t = i_t^* + \delta\widehat{b}_t - E_t \pi_{t+1} - \widehat{r}_t^n$$

14. Real interest rate

$$r_t = i_t^* + \delta\widehat{b}_t - E_t \pi_{t+1}$$

# 4 Simulation

## 4.1 Calibration

The calibration for the model is shown in table 1 below:

Table 1: Calibration

| Parameter | Symbol | Value | Source |
|---|---|---|---|
| Discount factor | $\beta$ | 0.91 | Berg et al, (2012) |
| Coefficient on marginal cost in the philips curve | $\lambda$ | 0.1058 | Berg et al, (2012) |
| Government expenditure/GDP | $\chi$ | 0.25 | ZIM Treasury |
| Fractions of firms allowed to change prices | $\theta$ | 0.75 | Gali and Monacelli, (2008) |
| Contrbution of foreign inflation in CPI | $\alpha$ | 0.6 | ZIMSTAT |
| Dixit-Stiglitz parameter for within-sector consumption | $\epsilon$ | 6 | Gali and Monacelli, (2008) |
| Elasticity of labour supply | $\varphi$ | 3 | Gali and Monacelli, (2008) |
| Steady state markup | $\mu$ | 1.2 | Gali and Monacelli, (2008) |
| Coefficient of US dollar demand gap | $\alpha_m$ | 0.501 | Simulation |
| Elasticity of government bonds | $\delta$ | 0.061 | Simulation |
| Elasticity of money demand | $\eta$ | 4 | Simulation |
| **Persistence parameters:** | | | |
| Persistance of government spending | $\rho^b$ | 0.7 | Simulation |
| Persistance of money supply | $\rho^{ms\$}$ | 0.90 | Simulation |
| Persistance of productivity | $\rho^a$ | 0.4975 | Peiris and Saxegaard (2007) |
| Persistance of foreign inflation | $\rho^{p_{f,t}}$ | 0.60 | Peiris and Saxegaard (2007) |
| Persistance of foreign interest rate | $\rho_m$ | 0.66 | Peiris and Saxegaard (2007) |

The discount factor takes the value, $\beta = 0.91$ to reflect steady state annual average interest rates of 10 percent per annum. The coefficient of marginal cost in the philips curve is given by $\lambda = 0.1058$ consistent with the discount factor see Berg, (2010). The ratio of government spending as a percentage of GDP in Zimbababwe is given $\chi = 0.25$. We let $\varphi = 3$ to represent labour elasticity of $\frac{1}{3}$, calvo parameter, $\theta = 0.75$ represents the fraction of firms allowed to change prices. We assume a steady state markup of $\mu = 1.2$ . We set substitution parameter between differenciated goods, $\epsilon = 6$, following Gali and Monacelli, (2008). Nominal Elasticity of US dollar demand $\eta = 4$ is close to 3 used in standard literature. The coefficient of US dollar demand gap, $\alpha_m = 0.501$ see Birchamol, (2014). We assume the government heavily relies on bonds to finance its deficit, $\rho^b = 0.70$. Productivity is low in subsaharan africa, and we use a persistance of productivity parameter of $\rho^a = 0.495$ see Peiris and Saxegaard (2007). Foreign interest rate and inflation shock persistance parameters are assigned the following values: $\rho^{i_t^*} = 0.60$ and $\rho^{p_{f,t}} = 0.60$ as proposed by Peiris and Saxegaard (2007). US dollar supply is an exogenous process. We assume $\rho^{ms\$} = 0.90$ consistent with standard DSGE literature.

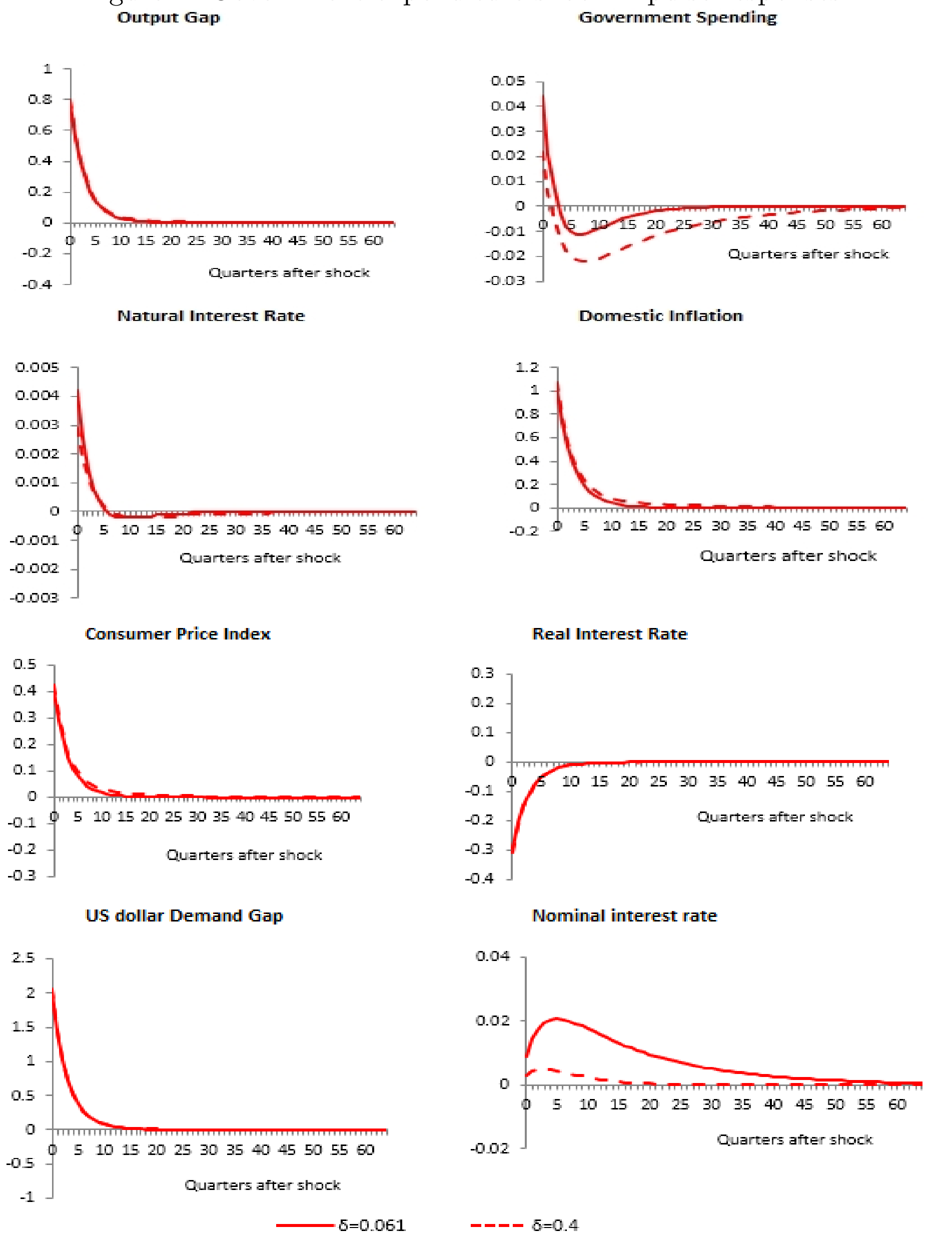
## 4.2 Government expenditure shock

Using small values of the sovereign risk premium parameter, fiscal expansion increases inflation. Given that monetary policy is constrained to increase norminal interets rates to contain inflation, real interest rate decline causing output to increase. This result is consistent with standard literature for the demand side transmission of fiscal policy see Christiano, Eichenbaum, and Rebelo (2011); Eggertsson (2011); and Woodford (2011). On the other had given a large increase in the sovereign risk premium parameter, fiscal policy has to substantially contract in order to reduce norminal and real interest rates. This also results in output expansion indicating the dominance of sovereign risk premium over the demand channel for the transmission of fiscal policy. These results are consistent with Contractionary Fiscal Policy literature popularised by Giavazzi and Pagano (1990), Blanchard, (1990) and Alesina and Perotti (1997) and Alesina and Ardagna (1998) and Corsetti, ( 2013). In summary, we believe our model offers a correct description of the stylised facts of the Zimbabwean economy. Table 1 shows the model baseline calibration and the impulse responses for the government expenditure shock are shown in Figure 1.

Table 1: Baseline calibration: $0.061 \leq \delta \leq 0.4$

| **Fiscal policy parameter** | **Symbol** | **Simple Policy** |
|---|---|---|
| Government bond | $\phi_{\widehat{d}}$ | 0.01 |
| Inflation | $\phi_{\pi}$ | 0.05 |
| Output | $\phi_{\widehat{y}}$ | 1.2 |

Figure 1: Government expenditure shock impulse responses



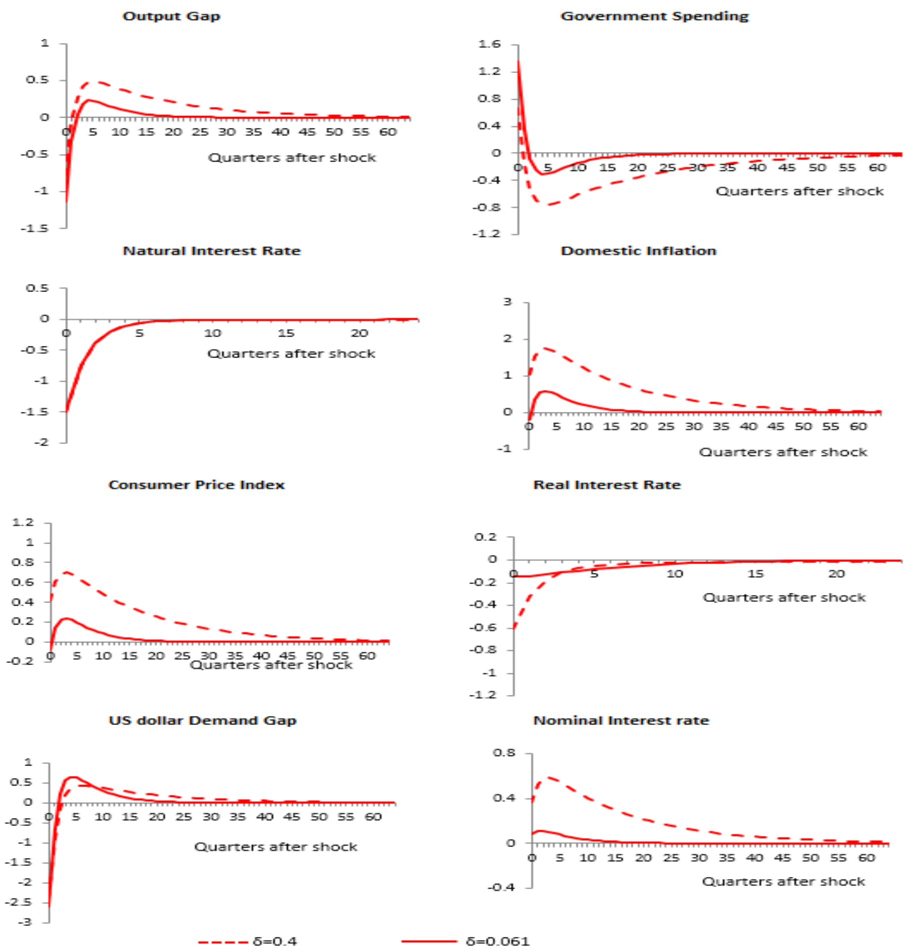
## 4.3 Productivity shock

A productivity shock increases the natural level of output. Increases in marginal costs cause inflation to rise. Fiscal policy immidiately expands to accomodate an increase in output and this further excercerbates inflationary pressures and reduces the real interest rate. Fiscal policy gradually contracts thereafter causing an easing-off in inflationary pressues and a rise in real interest rate until the economy returns to equilibrium see also Gali and Monacelli, (2008). When the risk premium parameter is large fiscal policy actions cause output to overshoot. The baseline calibration is shown in Table 1. We show impulse responses in Figure 2.

Table 2: Baseline calibration: $0.061 \leq \delta \leq 0.4$

| **Fiscal policy parameter** | **Symbol** | **Simple Policy** |
|---|---|---|
| Government bond | $\phi_{\widehat{d}}$ | 0.01 |
| Inflation | $\phi_{\pi}$ | 0.05 |
| Output | $\phi_{\widehat{y}}$ | 1.2 |

Figure 2: Productivity shock impulse responses


Output Gap
Quarters after shock
Government Spending
Quarters after shock
Natural Interest Rate
Quarters after shock
Domestic Inflation
Quarters after shock
Consumer Price Index
Quarters after shock
Real Interest Rate
Quarters after shock
US dollar Demand Gap
Quarters after shock
Nominal Interest rate
Quarters after shock
δ=0.4
δ=0.061

## 4.4 Foreign interest rate shock

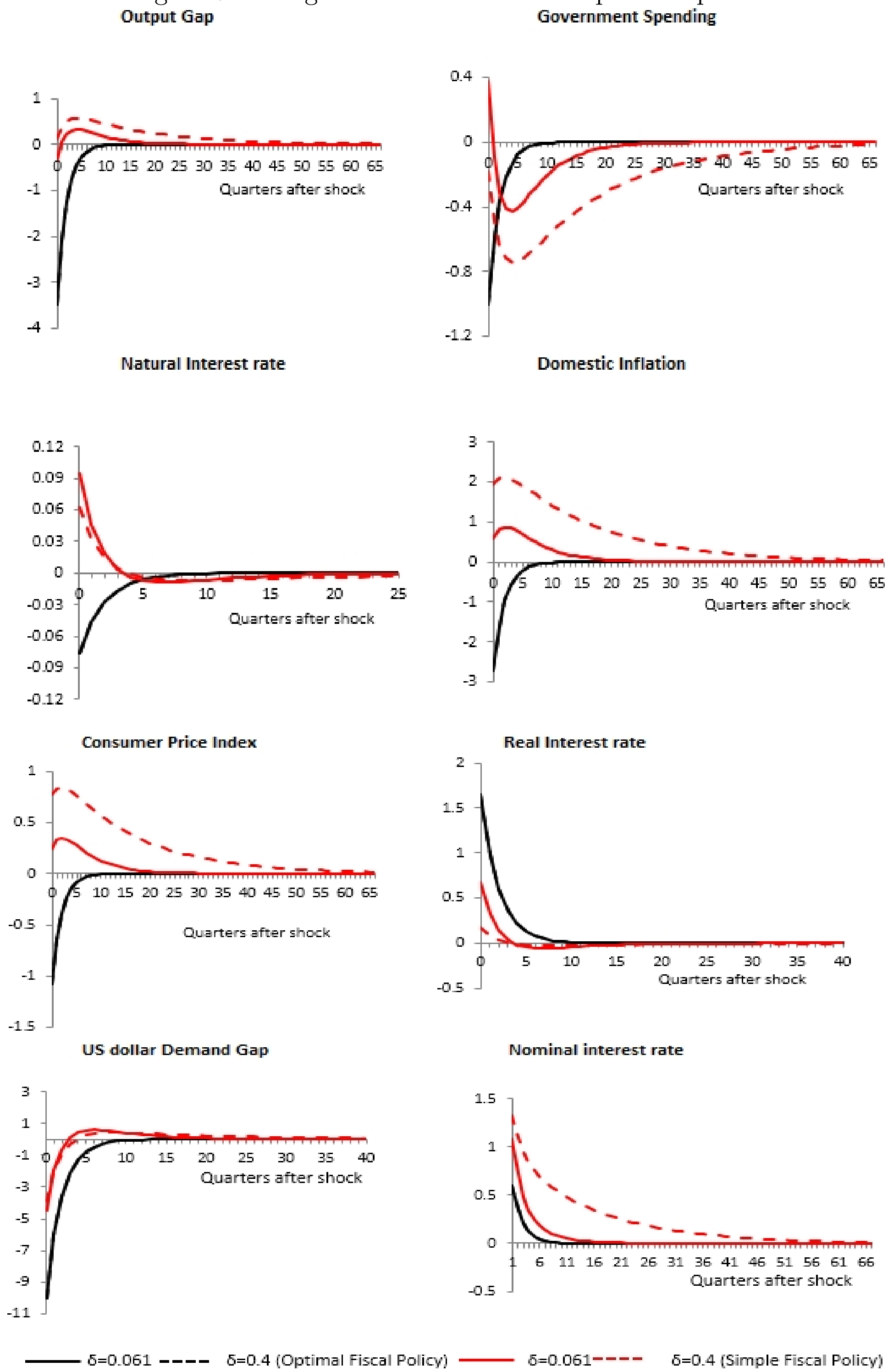
Optimal fiscal policy maximises the welfare function of economic agents facing an interest rate shock and an increase in the sovereign risk premium. Simple and optimal simple fiscal policies are countercyclical and procyclical as shown in Table 4. The optimal fiscal policy loss function is very small an indication that procyclical policy efficiently stabilises the economy.

Table 4: Foreign interest shock: Simple and Optimal simple fiscal policy rules

| **Parameter** | **symbol** | **Simple Policy** | **Simple Policy** | **Optimal policy** | **Optimal policy** |
|---|---|---|---|---|---|
| Sovereign risk | $\delta$ | 0.061 | 0.4 | 0.061 | 0.4 |
| **Fiscal rule** | | | | | |
| Government bond | $\phi_{\widehat{d}}$ | 0.01 | 0.01 | -2.37 | -1.98 |
| Inflation | $\phi_{\pi}$ | 0.05 | 0.05 | -2.90 | -0.91 |
| Output | $\phi_{\widehat{y}}$ | 1.2 | 1.2 | 0.61 | -0.003 |
| Social Loss Function | | | | 9.28e-011 | 1.04e-013 |

We find that a foreign interest rate shock should be accomodated by fiscal consolidation. This is consistent with suggestions by Corchrane, (2014). Precisely, fiscal contraction stabilises the economy by reducing the risk premium which causes nominal and real interest rates to decline and this positively impacts on output. Procyclical fiscal policy stabilises the economy faster and reduces the volatility in inflation and output. We also find that large fiscal contractions are expansionary. Our results are consistent with Corseti, (2013). Figure 4 shows impulse response functions for a foreign interest rate shock.

Figure 3: Foreign interest rate shock impulse responses



### 4.5 US dollar supply shock

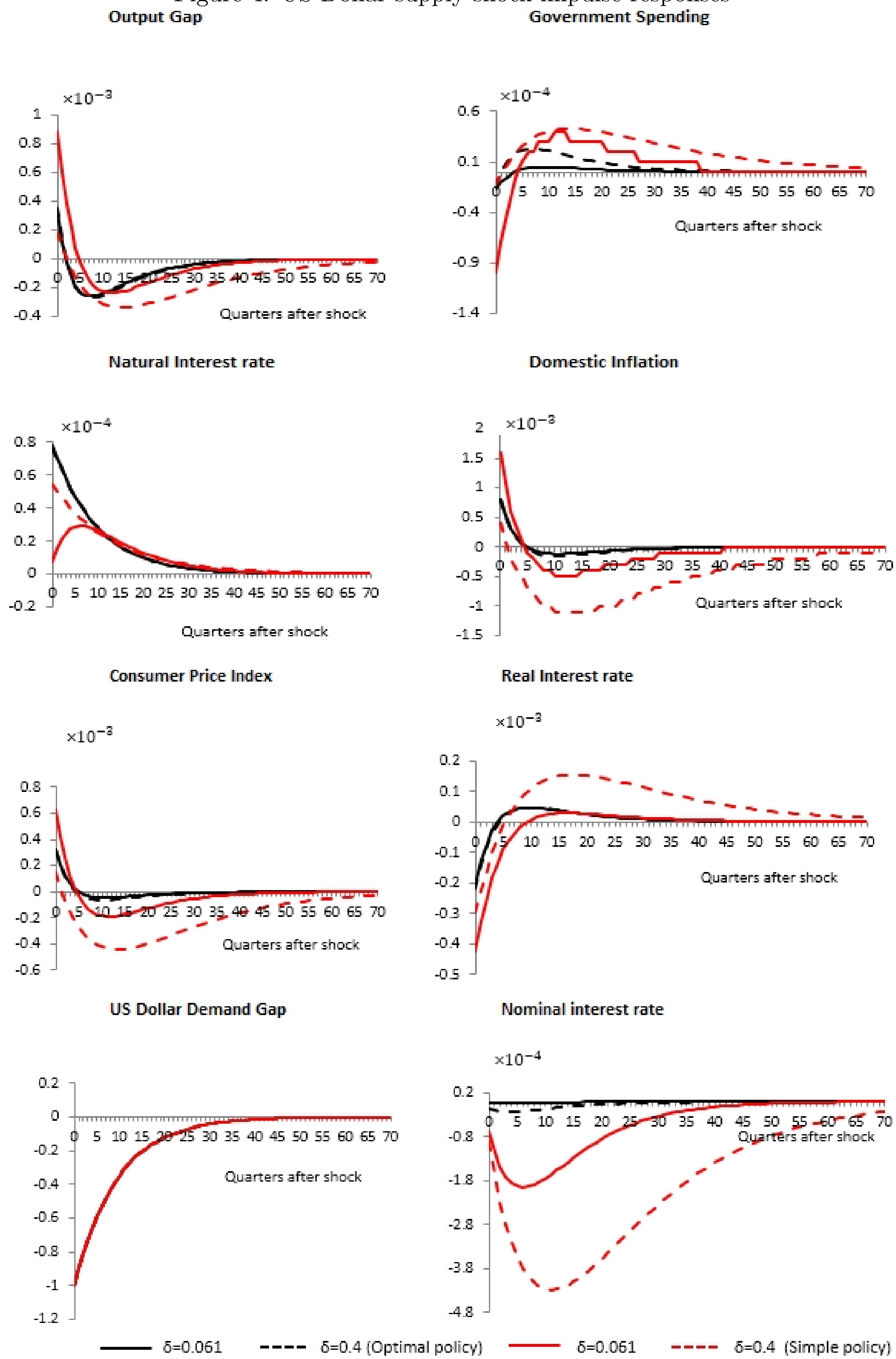
A dollarised economy facing US dollar supply shock and changes in the sovereign risk premium can be optimally stabilised by procyclical fiscal policy. Optimal fiscal policy effectively stabilises the economy by minimising the social loss function as shown in Table 5 below.

Table 4: US Dollar supply shock: Simple and Optimal simple fiscal policy rules

| **Parameter** | **symbol** | **Simple Policy** | **Simple Policy** | **Optimal policy** | **Optimal policy** |
|---|---|---|---|---|---|
| Sovereign risk | $\delta$ | 0.061 | 0.4 | 0.061 | 0.4 |
| **Fiscal rule** | | | | | |
| Government bond | $\phi_{\widehat{d}}$ | 0.01 | 0.01 | -1.95 | -1.87 |
| Inflation | $\phi_{\pi}$ | 0.05 | 0.05 | -0.54 | -0.71 |
| Output | $\phi_{\widehat{y}}$ | 1.2 | 1.2 | 0.44 | 0.62 |
| Objective function | | | | 4.56e-007 | 4.27e-007 |

A shock to US Dollar inflows is analogous to an unanticipated increase in US dollars into the dollarised economy from the rest of the world economy. Inflation increases and real interest rates to decline leading to output expansion. The impact of the shock is, however, very small Our study suggests that quantity of money in the dollarised economy can explain fluctuations in output and inflation. This is theoretically plausible and consistent with findings of Canova, (2011) Figure 4 below shows that procyclical policy stabilises the economy efficiently.

Figure 4: US Dollar supply shock impulse responses



### 4.6 Foreign inflation shock

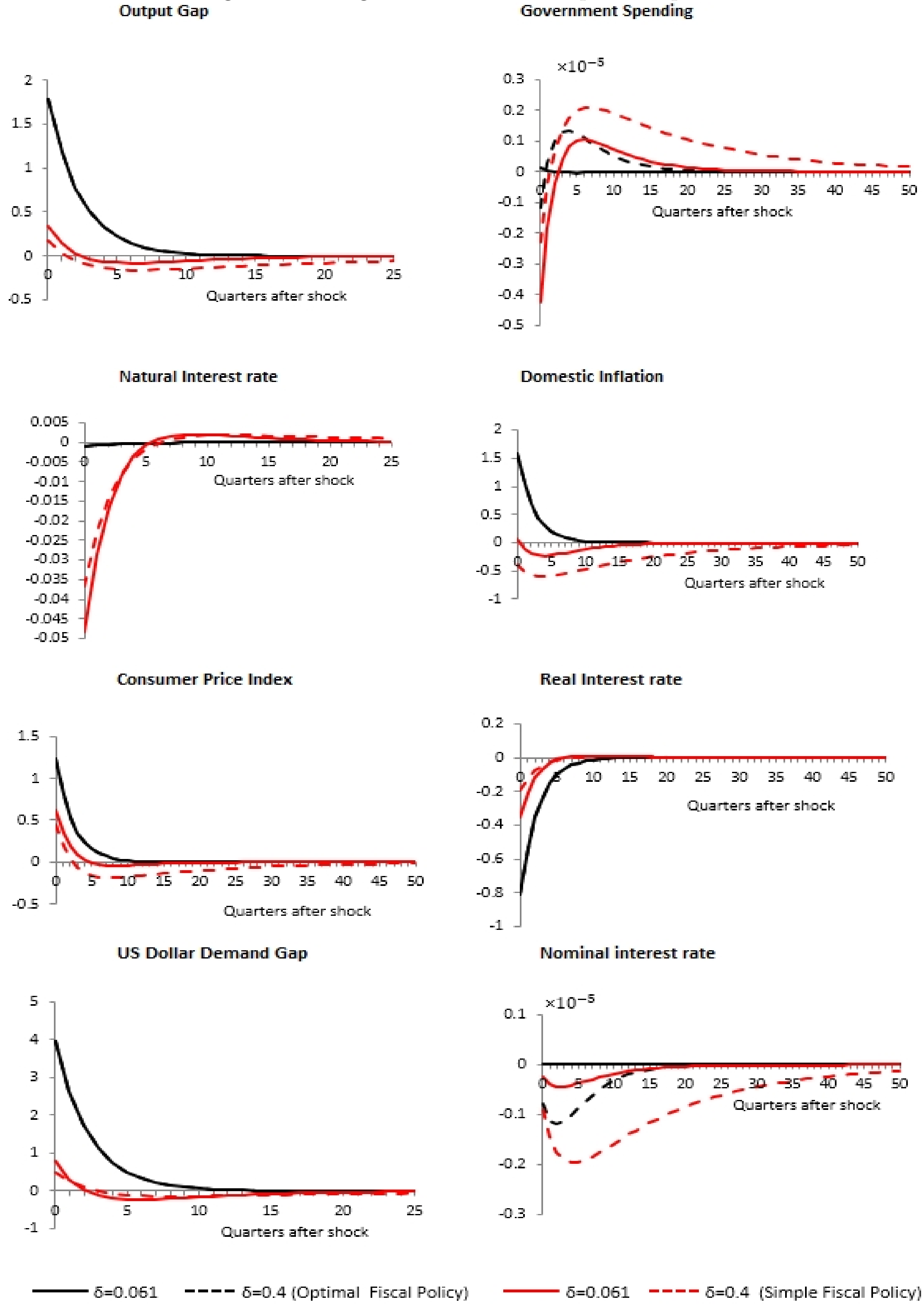
Procyclical fiscal policy stabilises a dollarised economy facing an foreign inflation shock and an increase in the sovereign risk premium.Table 5 below shows the simple and optimal fiscal policy rules.

Table 5: Foreign inflation shock: Simple and Optimal Simple fiscal policy rules

| **Parameter** | **symbol** | **Simple Policy** | **Simple Policy** | **Optimal policy** | **Optimal policy** |
|---|---|---|---|---|---|
| Sovereign risk | $\delta$ | 0.061 | 0.4 | 0.061 | 0.4 |
| **Fiscal rule** | | | | | |
| Government bond | $\phi_{\widehat{d}}$ | 0.01 | 0.01 | -1.85 | -1.66 |
| Inflation | $\phi_{\pi}$ | 0.05 | 0.05 | -0.33 | -0.77 |
| Output | $\phi_{\widehat{y}}$ | 1.2 | 1.2 | 0.23 | 0.53 |
| Objective function | | | | 1.81e-013 | 8.64e-012 |

A foreign inflation shock is expansionary. The incease in inflation cannot be contained by norminal interest rate increase since monetary policy is constrained. Optimal fiscal policy requires fiscal policy to marginaly expand. This action is not sufficient to contain inflation persistance and this reduces the real interest rate and causes output to expand. Our findings are consistent with proposals in Eggertson, (2010). Impulse responses are shown in Figure 5 below.

Figure 5: Foreign inflation shock impulse responses



# 5 Conclusion

Our paper investigates conduct of fiscal policy in a dollarised economy with a sovereign risk premium transmission channel for fiscal policy. We assume transmission of fiscal policy through the sovereign risk premium channel. Households seek compensation for holding government bonds by charging a sovereign risk premium over and above the world interest rate.and this affects firm funding conditions. An increase in government bonds increases the sovereign risk premium and the reverse holds. We use a standard New Keynesian DSGE model developed for a dollarised economy.to investigate the stability dynamics of simple and optimal simple fiscal policy rules.

Using baseline calibration, we investigate the the impulse responses of government spending and productivity shocks.Fiscal expansion results in an increase in output, domestic inflation, money demand, norminal interest rate and a decrease in real interest rate. We also find that fiscal contraction lowers the sovereign risk premium leading to output expansion.These results are consistent with keynesian and non keynesian effects of fiscal policy. A productivity shock reduces the real interest rate leading to an increase in the natural output level. Fiscal policy needs to expand in order to accomodate additional output increase as predicted by economic theory. Further, for large values of the sovereign risk premium parameter, fiscal stabilisation following a technology shock results in ouput overshooting.

We find that an increase in sovereign risk premium worsens macroeconomic conditions for a dollarised economy through its effect on increasing norminal and real interest rates resulting in a in consumption and output contraction. Simple optimal fiscal policy is procyclical and stabilises the economy efficiently. Precisely, stability time horizon is shorter and the policy mitigates volitities in inflation and output.

A foreign interest rate shock should be accomodated by fiscal consolidation. Fiscal consolidation leads to reduction in the sovereign risk premium and this lowers overal funding conditions in the economy. Our study find that a foreign inflation shock is expansionary. Precisely, an increase in foreign inflation when monetary policy is constrained leads to a reduction in the real interest rate and this results in consumption and output expansion. Another additional finding is that a US dollar supply shock has no significant impact on real variables. The quantity of US dollars in the economy, however, can explain fluctuations in inflation and output in a dollarised economy.

Our results have important policy implications. Optimal fiscal policy for a dollarised economy with a sovereign risk channel for fiscal policy is procyclical. A big question that emerges is: Has fiscal policy been effective in stabilising the Zimbabwean economy under dollarisation? We leave this for further research.

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## APPENDIX A

### Households

The representative households are infinetly lived and supply labour and capital to manufacturing companies. Households are constrained by their budget but seek to maximise the following expected lifetime utility:

$$E_t \sum_{t=0}^{\infty} \beta U(C_t, N_t, G_t, M\$_t) \tag{utility}$$

The utility function $U(C_t, N_t, G_t)$ given by

$$U(C_t, N_t, G_t) = (1-\chi)\log C_t + \chi \log(G_t) - \frac{{N_t}^{1-\varphi}}{1-\varphi} + \frac{M\$_t}{P_t} \tag{A1}$$

where $E_t$ is conditional on time t, $\beta \in [0,1]$ is the discount factor, $G_t$ is an index for Government consumption, $N_t$ is labour supply normalised to 1 $0 \leq N_t \leq 1$ , $\chi$ is relative elastisticity of government consumption relative to consumption in the private sector, $M\$_t$ represents foreign currency holdings and $\varphi$ is marginal elasticity of labour supply. The consumption index of domestic produced goods and imports for the small open economy is represented by $C_t$ as shown below.

$$C_t \equiv \frac{(C_{d,t})^{1-\alpha}(C_{f,t})^{\alpha}}{(1-\alpha)^{1-\alpha}(\alpha)^{\alpha}}$$

The measures of elastisticity of substitution of domestic goods for foreign goods and the openness of an economy are given by $\alpha$ for $0 \leq \alpha \leq 1$ and $\upsilon$ (strictly greater than zero) is the elasticity of substitution for locally produced goods and imports from the rest of the world within the domestic economy. Elaticities of substitution indicies domestic goods $C_{d,t}$ and imported goods $C_{f,t}$ are represented as follows:

$$C_{d,t} \equiv \left( \int_0^1 C_{d,t}(f)^{\frac{\varepsilon-1}{\varepsilon}} df \right)^{\frac{\varepsilon-1}{\varepsilon}}, C_{f,t} \equiv \left( \int_0^1 C_{f,t}(f)^{\frac{\varepsilon-1}{\varepsilon}} df \right)^{\frac{\varepsilon-1}{\varepsilon}}$$

where, $\varepsilon$ represents economy wide elasticity of substitution for a busket of goods for domestic and foreign produced goods. Demand utility functions for the domestic households is given by the following:

$$C_{d,t}(j) \equiv \left( \frac{P_{d,t}(j)}{P_{d,t}} \right)^{-\varepsilon} C_{d,t}, C_{f,t}(j) \equiv \left( \frac{P_{f,t}(j)}{P_{f,t}} \right)^{-\varepsilon} C_{f,t}$$

Where, $P_{d,t}(j)$ and $P_{f,t}(j)$ are prices indices of various goods $j$ , produced by domestic and rest of the world producers, respectively. Domestic and rest of the world price indices are given by:

$$P_{d,t} \equiv \left( \int_0^1 P_{d,t}\left(j\right)^{1-\varepsilon} df \right)^{\frac{1}{1-\varepsilon}}, P_{f,t} \equiv \left( \int_0^1 P_{f,t}\left(j\right)^{1-\varepsilon} df \right)^{\frac{1}{1-\varepsilon}}$$

Using some straight forward algebra we represent the demand functions for locally manufactured products and imports in the domestic economy as shown below

$$C_{d,t} \equiv (1-\sigma)\left(\frac{P_{d,t}}{P_t}\right)^{-\varepsilon} C_t, C_{f,t} \equiv \sigma\left(\frac{P_{f,t}}{P_t}\right)^{-\varepsilon} C_t$$

where $P_t$ represents the aggregate Consumer Price Index for the small open economy and $\sigma$ is the proportion of total domestic expenditure used for import good consumption. We represent the household consumer price index as follows:

$$P_t = ((1-\alpha)P_{d,t}{}^{1-\varepsilon} + \alpha P_{f,t}{}^{1-\varepsilon})^{\frac{1}{1-\varepsilon}} \tag{A2}$$

We let $\varepsilon = 1$, and wite $P_t = P_{d,t}{}^{1-\alpha}(P_t{}^*)^{\alpha}$, and assume $P_t^* = P_{f,t}$ is the rest of the world economy consumer price index. The representations of consumption of domestic and imports as shown below:

$$P_f C_{f,t} {=} \alpha P_{d,t} C_t$$

$$P_t C_t = (1-\alpha) P_{d,t} C_t$$

The household consumption in a small open economy is given by:

$$P_{d,t}C_{d,t} + P_{f,t}C_{f,t} = P_t C_t \tag{A3}$$

The households seek to maximise their utility subject to the following budget constraint:

$$P_t C_t + M\$_t {+} E_t \left\{ \left[ (1+i_t^*)\Psi(B_t) \right]^{-1} B_t \right\} + T = B_t {+} W_t N_t + M\$_{t-1} - T \tag{A4}$$

where $P_t$ is the represents aggregate price of consumed goods in the domestic economy, $\Psi(B_t)$ is the risk premium on government bonds, $B_t$ denotes bond holdings at time period t, $W_t$ represents wages, $N_t$ is labour, $\Pi_t$ donotes firm profits, $M\$_t$ denotes US dollar holdings, and $T_t$ denotes lumpsum government tax collections. Households earn income from wages, profits from firms and return, $(1+r_t)\Psi(B_t)$, from holding government bonds, which they charge a risk premium in addition to the norminal interest rate.

### 5.0.1 Risk Premium

The risk premium,$\Psi(B_t)$ is a an increasing function of government debt, grosss domestic product and is represented as follows

$$\Psi(B_t) = \delta B_t \tag{A5}$$

The intuition is that households charge a premium as an insurence against unforeseen losses in government bonds else households would prefer to keep money in the bank or under their pillows.

First order conditions for the small open economy household problem are given by:

$$\frac{W_t}{P_{d,t}} = \frac{C_t N_t^{\varphi}}{1-\chi} \tag{A6}$$

The household labour supply decisions are influenced by lumpsum government taxes and preferences for government produced goods.

$$\beta(1+i_t^*)\Psi(B_t)E_t\left\{\frac{C_t}{C_{t+1}}\frac{P_{d,t}}{P_{d,t+1}}\right\} = 1 \tag{A7}$$

where $R_t$ represents the an exogenous interest rate plus a default risk premium on government bonds.

The marginal utility of holding foreign currency balances for households is given by the following

$$\frac{\frac{M_t}{P_t}}{C_t} = \frac{R_t}{1+R_t} \tag{A8}$$

log linearising we get

$$w_t = c_t + \varphi n_t - \log(1-\chi) \tag{A9}$$

Where equation (A8), represents the the labour supply equation.

$$c_t = E_t c_{t+1} - (i_t^* + \delta\widehat{b}_t - E_t\pi_{t+1} - \rho) \tag{A10}$$

where $\delta\widehat{b_t}$ is the bond risk premium and $\delta$ is the coefficient of government bond stock elasticity,$\rho = -\log\beta$ is equilibrium real interest rate and $i_t^*$ is the norminal interest rate from the rest of the world.

$$md\$_t = p_t + c_t - \eta(i_t^* + \delta\widehat{b_t}) \tag{A11}$$

where, $md\$_t$ represents the demand for US dollars in the economy. Intutively equation (A11) implies that the relationship between money and inflation holds in equilibrium if consumption and interest rates are independent of US dollar holdings. In order to solve the model, US dollar demand should equal to US dollar supply. The US dollars are supplied by the from the rest of the world economy. We represent US dollar supply as a shock process.as shown below

$$ms\$_t = \rho^{ms\$} ms_{t-1} + \varepsilon^{ms\$} \tag{A12}$$

**The Productive Sectors**

We assume there exists a continuum of firms that operate in monopolistic business environment.

*Firms*

Firms produce differenciated goods. The money- in- the production for firm $j$ is a variant of one proposed by Benchimol, (2014) and is given by

$$Y_t(f) = \left(\frac{M_t}{P_t}(j)\right)^{\alpha_m} A_t(j) N_t(j) \tag{A13}$$

The firms aggregate production function is represented as follows

$$Y_t = \left(\frac{M\$_t}{P_t}\right)^{\alpha_m} A_t N_t \tag{A14}$$

where $A_t$ denotes country wide firm firm productivity parameter, $Y_t$ is firm output and $N_t$ represents labour force. The condition for market clearing for labour is given

by $\int_0^1 N_t(j)d(i)$.

$$Y_t(f) = \int_0^1 \frac{Y_t(j)}{A_t(f)\left(\frac{M\$_t}{P_t}(j)\right)^{\alpha_m}} d(j) \tag{A15}$$

$$Y_t(f) = \frac{Y_t(j)}{A_t(f)\left(\frac{M\$_t}{P_t}(j)\right)^{\alpha_m}} \int_0^1 \frac{P_t(f)}{P_t} d(j) \tag{A16}$$

Log linearising we get

$$y_t = a_t{+}n_t{+}\alpha_m m\$ + d_t \tag{A17}$$

where, $a_t = \rho^a a_{t-1} + \varepsilon_t^a$ is the technology shock for all the firms,$m$ is the money demand gap which we define as money demad minus money supply, $\alpha_m$ is the coeeficient of the money demand gap, $d_t = \log\left(\int_0^1 \frac{P_t(j)}{P_t} d(j)\right) = 0$ according to Gali, (2008) and $d(j)$ represents firm wide price desparities. We then obtain

$$y_t = a_t{+}n_t{+}\alpha_m m\$ \tag{A18}$$

We add the price stickiness concept popularised by calvo (1983) and denote $\omega$ to represent the probability of a firm not changing prices at a given time period, we set $1 - \omega$ to represent the fraction of firms that can optimally change prices at a given time perid. Firms allowed to change prices set their optimal prices to prices to $\overline{p}_t$. The optimised price selected by the firms allowed to change prices at time $t$ seek to maximise the following expected profit function:

$$E_t \left\{ \sum_{x=0}^{\infty} \omega^x R_{t,t+x} Y_{t+x} (\overline{P}_t - \frac{\rho}{1-\rho} P_{t+x} MC_{t+x}) \right\} = 0 \tag{A19}$$

where, $\varepsilon/(\varepsilon - 1)$ denotes the inovation to price markup, $MC_t^n$ represents price markup, and $R_{t,t+x}$ represents the return of a risk free security.

Using the Dixit-Stiglitz (1977) aggregator, we represent firm aggregate price index as shown in equation 9:

$$p_t = \left((1-\omega)\left(\overline{p}\right)^{1-\epsilon} + \omega(p_t)^{1-\epsilon}\right)^{\frac{1}{1-\epsilon}} \tag{A20}$$

Using the dictacts of law of motion we represent $p$ as follows

$$p_t = (1 - \omega(\overline{p}_t) + \omega(p_{t-1}) \tag{A21}$$

We write the optimal price chosen my firms that are allowed to change prices as:

$$\overline{p}_t = \mu + (1 - \beta\omega) \sum_{x=0}^{\infty} (\beta\omega)^x E_t(mc_{t+x}{+}p_t) \tag{A22}$$

The intution is that firms that are allowed to change prices would choose a adjust prices would set prices after considering current prices and marginal cost as well as future marginal costs. Inflation would increase if the firms's steady state markups are below the anticipated average markups. Precisely, firms would want to set prices that are above average to maximise their profits.

Domestic inflation will therefore be given by sum of expected future inflation and expected real marginal costs as shown below;

$$\pi_t = \beta E_t \left\{\pi_{t+1}\right\} + \lambda \widehat{mc}_t \tag{A23}$$

where, $\lambda = \frac{(1-\beta\omega)(1-\omega)}{\omega}$ and $\widehat{mc}_t$ is the real marginal cost for firms.

### 5.0.2

### Market Clearing Condition

Market clearing implies that all the produced products are consumed. The equilibrium condition is given by:

$$y_t = \chi g_t + (1-\chi)c_t$$

$$c_t = \frac{1}{(1-\chi)}y_t - \frac{\chi}{(1-\chi)}g_t \tag{A24}$$

**Marginal Cost for firms**

We derive the natural rate of interest for the dollarised economy. The relation between the real marginal cost for firms and relative gain for hiring more labour is given as follows:

$$MC_t^n = \frac{W_t}{P_t MPN_t^n m\$_t^n}(1-\chi) \tag{A25}$$

Loglinearising we obtain

$$mc_t^n = w_t - p_t - mpn_t^n - mpm\$_t^n + \log(1-\chi) \tag{A26}$$

We differentiate equation (A26) and take the logarithim of the differentiated fuction to find a representation for the marginal productivity associated with of adding a unit of labour to increase production as shown below:

$$mpn_t^n = a_t + \alpha_m m\$_t \tag{A27}$$

Differentiating (A26) with respect to money demand gap and take the logarithim of the differentiated fuction, we obtain an expression for real marginal cost for money demand represented as follows

$$mpm_t^n = a_t + \log \alpha_m + (\alpha_m - 1)m\$_t + n_t \tag{A28}$$

Substituting equation (A27) into equation (A26), we obtain the following expression for marginal cost for firms:

$$mc_t^n = (w_t - p_t) - mpn_t^n - mpm\$_t^n + \log(1-\chi)$$

$$mc_t^n = c_t + \varphi n_t - \log(1-\chi) - a_t - \alpha_m m\$_t - a_t - \log \alpha_m - (\alpha_m - 1)m\$_t - n_t$$

$$mc_t^n = c_t + n_t(\varphi - 1) - \log(1-\chi) - 2a_t - (2\alpha_m - 1)m\$_t - \log \alpha_m \tag{A29}$$

We substitute the representation for consumption in equation (A24) in (A29) and obtain the following alternative represenation

$$mc_t^n = \frac{1}{(1-\chi)} y_t - \frac{\chi}{(1-\chi)} g_t + (\varphi - 1)y_t - (\varphi - 1)a_t - 2a_t - (2\alpha_m - 1)m\$_t - \log(1-\chi)$$

$$mc_t^n = \frac{(1 + (\varphi - 1)(1-\chi))}{(1-\chi)} y_t - \frac{\chi}{(1-\chi)} g_t - a_t(\varphi + 1) - (2\alpha_m - 1)m\$_t - \log(1-\chi) \tag{A30}$$

We denote $y_t^n$ and $g_t^n$, as natural output level and natural government expenditure as equilibrium levels under fully flexible prices. We substitute $y_t^n$ and $g_t^n$ inequation (A30)

and obtain the following expression for the desired markup, $mc^n$ for firms.

$$mc^n = \frac{(1 + (\varphi - 1)(1-\chi))}{(1-\chi)} y_t^n - \frac{\chi}{(1-\chi)} g_t^n - a_t(\varphi + 1) - (2\alpha_m - 1)m\$_t^n - \log(1-\chi) \tag{A31}$$

We subtract equation (A31) from equation (A30) to get the real marginal cost for firms as shown bellow.

$$\widehat{mc}_t = mc_t^n - mc^n = \frac{(1+(\varphi-1)(1-\chi))}{(1-\chi)} y_t - \frac{\chi}{(1-\chi)} g_t - a_t(\varphi+1) - (2\alpha_m - 1)m\$_t - \log(1-\chi)$$
$$- \left( \frac{(1+\varphi(1-\chi))}{(1-\chi)} y_t^n - \frac{\chi}{(1-\chi)} g_t^n - a_t(\varphi+1) - (2\alpha_m - 1)m\$_t^n - \log(1-\chi) \right)$$

Rearranging and collecting like terms

$$\widehat{mc}_t = mc_t^n - mc^n = \frac{(1+(\varphi-1)(1-\chi))}{(1-\chi)} (y_t - y_t^n) - \frac{\chi}{(1-\chi)} (g_t - g_t^n) - (2\alpha_m - 1)(m\$_t - m\$_t^n)$$
$$-a_t(\varphi+1) + a_t(\varphi+1) - \log(1-\chi) + \log(1-\chi)$$

By further simplifying we obtain the following:

$$\widehat{mc}_t = \frac{(1+(\varphi-1)(1-\chi))}{(1-\chi)} (y_t - y_t^n) - \frac{\chi}{(1-\chi)} (g_t - g_t^n) - (2\alpha_m - 1)(m\$_t - m\$_t^n)$$

Representing the real marginal cost in gap form we obtain

$$\widehat{mc}_t = \frac{(1+(\varphi-1)(1-\chi))}{(1-\chi)} \widehat{y}_t - \frac{\chi}{(1-\chi)} \widehat{g}_t - (2\alpha_m - 1)\widehat{m\$}_t \tag{A32}$$

Real marginal cost is negatively related to government spending and money demand gap $(md\$_t - ms\$_t)$. For a givel output level, government spending crowds out domestic consumption. The other explanation is that an increase in government spending causes an appreciation and reduces the real wage.

**New Keynesian Philips curve**

The standard New Keynesian Philips curve notation is represented as follows:

$$\pi_{d,t} = \beta E_t \pi_{t+1} + \lambda \widehat{mc}_t \tag{A33}$$

We substitute the marginal cost gap into the standard new philps curve to obtain the following representation for the dollarised economy.

$$\pi_{d,t} = \beta E_t \pi_{d,t+1} + \lambda \frac{(1+(\varphi-1)(1-\chi))}{(1-\chi)} \widehat{y}_t - \frac{\lambda\chi}{(1-\chi)} \widehat{g}_t - \lambda(2\alpha_m - 1)\widehat{m\$}_t \tag{A34}$$

**Dynamic IS curve**

We derive the Dynamic IS curve. Using (A10) and (A24) see also a trick by Eggertson, (2010) we derive the standard new keynesian IS curve

$$\frac{1}{(1-\chi)}y_t - \frac{\chi}{(1-\chi)}g_t = \frac{1}{(1-\chi)}E_t y_{t+1} - \frac{\chi}{(1-\chi)}E_t g_{t+1} - (i_t^* + \delta\widehat{b}_t + E_t\pi_{t+1} - \rho)$$

We rearrage by multiplying throughout by $(1-\chi)$ and make output, $y_t$ the subject of the formula as shown below

$$y_t - \chi g_t = E_t y_{t+1} - (1-\chi)(i_t^* + \delta\widehat{b}_t + E_t\pi_{t+1} - \rho) - \chi E_t g_{t+1}$$

We make output, $y_t$ the subject of the formula as shown below:

$$y_t = E_t y_{t+1} - (1-\chi)(i_t^* + \delta\widehat{b}_t + E_t\pi_{t+1} - \rho) + \chi g_t - \chi E_t g_{t+1}$$

Simplifying by factoring out $\chi$ we get:

$$y_t = E_t y_{t+1} - (1-\chi)(i_t^* + \delta\widehat{b}_t + E_t\pi_{t+1} - \rho) - \chi(E_t g_{t+1} - g_t)$$

Using identity $E_t\Delta g_{t+1} = (E_t g_{t+1} - g_t)$ obtain present output by the following expression:

$$y_t = E_t y_{t+1} - (1-\chi)(i_t^* + \delta\widehat{b}_t + E_t\pi_{t+1} - \rho) - \chi E_t\Delta g_{t+1} \tag{A35}$$

We represent real interest rate by, $r_t$:

$$r_t = i_t^* + \delta\widehat{b}_t - E_t\pi_{t+1} \tag{A36}$$

Inserting (A36) in (A35) we represent ouput as follows:

$$y_t = E_t y_{t+1} - (1-\chi)(r_t - \rho) - \chi E_t\Delta g_{t+1} \tag{A37}$$

Using the same steps we represent natural output, $y_t^n$, in terms of natural government spending, $g_t^n$, and natural interest rate interest rate, $r_t^n$, as shown below:

$$y_t^n = E_t y_{t+1}^n - (1-\chi)(r_t^n - \rho) - \chi E_t\Delta g_{t+1}^n \tag{A38}$$

We define output gap, $\widehat{y}_t$, as follows:

$$\widehat{y}_t = y_t - y_t^n \tag{A39}$$

Subtracting (A38) from (A37) we represent the dynamic IS in terms of the variables in their steady state

$$\widehat{y}_t = (E_t y_{t+1} - E_t y^n_{t+1}) - (1-\chi)(r_t - \rho + \rho - r^n_t) - \chi(E_t \Delta g_{t+1} - E_t \Delta g^n_{t+1}) \tag{A40}$$

We eliminate the equilibrium interest rate and obtain the following:

$$\widehat{y}_t = (E_t y_{t+1} - E_t y^n_{t+1}) - (1-\chi)(i^*_t + \delta \widehat{b}_t - E_t \pi_{t+1} - r^n_t) - \chi(E_t \Delta g_{t+1} - E_t \Delta g^n_{t+1}) \tag{A41}$$

Finally we simply and present the equation for output gap as follows:

$$\widehat{y}_t = E_t \widehat{y}_{t+1} - (1-\chi)(i^*_t + \delta \widehat{b}_t - E_t \pi_{t+1} - r^n_t) - \chi E_t \Delta \widehat{g}_{t+1} \tag{A42}$$

We apply the transversality condition $\lim_{T \to \infty} E_t y_{t+T} = \lim_{T \to \infty} E_t g_{t+T} = 0$ and solving (A42) we obtain

$$\widehat{y}_t = \chi \widehat{g}_t - (1-\chi)\left(E_t \sum_{k=0}^{\infty} (\widetilde{i}^*_{t+k} + \delta \widehat{b}_{t+1} - \pi_{t+k+1} - r^n_t)\right) \tag{A43}$$

Intutively equation (A43) tell us that the deviations in the is output gap will emanate from changes in summation of differences in the real interest rate and the natural interest rate consumption tax and deviations in government spending gap with the weights attached being, $\chi$ and $-(1-\chi)$,respectively.

**Natural Interest rate**

To derive the natural interest rate, we first assume flexible prices

$$-\mu = mc^n \tag{A44}$$

We substitute (A44) into equation (A29) we obtain

$$-\mu = c_t + n_t(\varphi - 1) - 2a_t - (2\alpha_m - 1)m\$_t - \log \alpha_m - \log(1-\chi) \tag{A45}$$

Eliminating $n_t$ we obtain

$$-\mu = c_t + \varphi(y_t - a_t) - 2a_t - (2\alpha_m - 1)m\$_t - \log(1-\chi) - \log \alpha_m$$

We also elimiminate consumption using equation (A24)

$$-\mu = \frac{1}{(1-\chi)}y_t - \frac{\chi}{(1-\chi)}g_t + (\varphi - 1)(y_t - a_t) - 2a_t - (2\alpha_m - 1)m\$_t - \log(1-\chi) - \log\alpha_m$$

Rearranging like terms we get

$$-\mu = \frac{(1+(\varphi-1)(1-\chi))}{(1-\chi)}y_t - \frac{\chi}{(1-\chi)}g_t - (\varphi+1)a_t - (2\alpha_m - 1)m\$_t - \log(1-\chi) - \log\alpha_m$$

Maing $y_t$ and its coefficient the subject of the formula we obtain

$$\frac{(1+(\varphi-1)(1-\chi))}{(1-\chi)}y_t = \frac{\chi}{(1-\chi)}g_t + (\varphi+1)a_t + (2\alpha_m - 1)m\$_t + \log(1-\chi) - \log\alpha_m - \mu$$

Simplifying and making $y_t$ the subject of the formula we remain with the following representation:

$$y_t = \frac{1-\chi}{(1+(\varphi-1)(1-\chi))}\left(\frac{\chi}{(1-\chi)}g_t + (\varphi+1)a_t + (2\alpha_m - 1)m\$_t + \log(1-\chi) - \log\alpha_m\right) + \frac{1-\chi}{(1+(\varphi-1)(1-\chi))}(\log(1-\chi) - \log\alpha_m - \mu) \quad \text{(A46)}$$

Using the usual notation $y_t^n$ and $g_t^n$ to represent natural output and natural government spending, respectively. We use equation (A46) we represent $y_t^n$ as a function of $g_t^n$

as shown below:

$$y_t^n = \frac{1-\chi}{(1+(\varphi-1)(1-\chi))}\left(\frac{\chi}{(1-\chi)}g_t^n + (\varphi+1)a_t + (2\alpha_m - 1)m\$_t^n\right) + \frac{1-\chi}{(1+(\varphi-1)(1-\chi))}(\log(1-\chi) - \log\alpha_m - \mu) \quad \text{(A47)}$$

Taking first differences of equation (A46):

$$E_t\Delta y_{t+1}^n = \frac{1-\chi}{(1+(\varphi-1(1-\chi))}\left(\frac{\chi}{(1-\chi)}E_t\Delta g_{t+1}^n + E_t\Delta(\varphi+1)a_{t+1} + \right) + \frac{1-\chi}{(1+(\varphi-1(1-\chi))}(E_t\Delta(2\alpha_m - 1)m\$_t^n + E_t\Delta\log(1-\chi) + E_t\Delta\mu) \quad \text{(A48)}$$

Simplifying equation (A43)

$$E_t\Delta y^n_{t+1} = \frac{(\varphi+1)(1-\chi)}{(1+(\varphi-1)(1-\chi))}E_t\Delta a_{t+1} + \frac{\chi}{(1+(\varphi-1)(1-\chi))}E_t\Delta g^n_{t+1} + \frac{(2\alpha_m-1)(1-\chi)}{(1+(\varphi-1)(1-\chi))}E_t\Delta m\$^n_{t+1} \quad \text{(A49)}$$

We rearrange equation (A37) to obtain the following

$$E_t\Delta y_{t+1} = (1-\chi)(r_t-\rho)+\chi E_t\Delta g_{t+1}$$

Substituting the real interest rate the representation becomes

$$E_t\Delta y_{t+1} = (1-\chi)(i^*_t+\delta\widehat{b}_t - E_t\pi_{t+1}-\rho)+\chi E_t\Delta g_{t+1} \quad \text{(A50)}$$

Using equation (A42) shown below

$$\widehat{y}_t = E_t\widehat{y}_{t+1} - (1-\chi)(i^*_t+\delta\widehat{b}_t - E_t\pi_{t+1} - r^n_t) - \chi E_t\Delta\widehat{g}_{t+1}$$

We make $r^n_t$, the subject of the formula

$$r^n_t = (i^*_t+\delta\widehat{b}_t - E_t\pi_{t+1}) - \frac{1}{(1-\chi)}(E_t\widehat{y}_{t+1}-\widehat{y}_t) + \frac{\chi}{(1-\chi)}E_t\Delta\widehat{g}_{t+1}$$

Expanding the represenation

$$r^n_t = (i^*_t+\delta\widehat{b}_t - E_t\pi_{t+1}) - \frac{1}{(1-\chi)}\left((E_ty_{t+1}-E_ty^n_{t+1})-(y_t-y^n_t)\right) + \frac{\chi}{(1-\chi)}E_t\Delta\widehat{g}_{t+1}$$

$$r^n_t = (i^*_t+\delta\widehat{b}_t - E_t\pi_{t+1}) - \frac{1}{(1-\chi)}\left((E_ty_{t+1}-y_t)-(E_ty^n_{t+1}-y^n_t)\right) + \frac{\chi}{(1-\chi)}(E_t\Delta g_{t+1}-E_t\Delta g^n_{t+1})$$

$$r^n_t = (i^*_t+\delta\widehat{b}_t - E_t\pi_{t+1}) - \frac{1}{(1-\chi)}\left(E_t\Delta y_{t+1}-E_t\Delta y^n_{t+1}\right) + \frac{\chi}{(1-\chi)}(E_t\Delta g_{t+1}-E_t\Delta g^n_{t+1})$$

Substituting equation (A32) we obtain the following

$$r^n_t = (i^*_t+\delta\widehat{b}_t - E_t\pi_{t+1}) - \frac{1}{(1-\chi)}\left((1-\chi)(r_t-\rho)+\chi E_t\Delta g_{t+1} - E_t\Delta y^n_{t+1}\right) + \frac{\chi}{(1-\chi)}E_t\Delta g_{t+1} - \frac{\chi}{(1-\chi)}E_t\Delta g^n_{t+1}$$

Rearranging and collecting like terms

$$\begin{aligned} r_t^n &= (i_t^* + \delta\widehat{b}_t - E_t\pi_{t+1}) - (i_t^* + \delta\widehat{b}_t - E_t\pi_{t+1} - \rho) - \frac{\chi}{(1-\chi)}E_t\Delta g_{t+1} \\ &+\frac{\chi}{(1-\chi)}E_t\Delta g_{t+1} + \frac{1}{(1-\chi)}E_t\Delta y_{t+1}^n - \frac{\chi}{(1-\chi)}E_t\Delta g_{t+1}^n \end{aligned}$$

Simplifying the expresssion

$$r_t^n = \rho + \frac{1}{(1-\chi)}E_t\Delta y_{t+1}^n - \frac{\chi}{(1-\chi)}E_t\Delta g_{t+1}^n$$

We substitute equation (A44) to the representation of the natural interest rate and obtain the following

$$\begin{aligned} r_t^n &= \rho + \frac{1}{(1-\chi)}\left(\frac{(\varphi+1)(1-\chi)}{(1+(\varphi-1)(1-\chi))}E_t\Delta a_{t+1} + \frac{\chi}{(1+(\varphi-1)(1-\chi))}E_t\Delta g_{t+1}^n\right) \\ &+\frac{1}{(1-\chi)}\frac{(2\alpha_m-1)(1-\chi)}{(1+(\varphi-1)(1-\chi))}E_t\Delta m\$_{t+1}^n - \frac{\chi}{(1-\chi)}E_t\Delta g_{t+1}^n \end{aligned}$$

Expanding by opening brackets we obtain

$$\begin{aligned} r_t^n &= \rho + \frac{\varphi+1}{(1+\varphi(1-\chi))}E_t\Delta a_{t+1} + \frac{\chi}{(1-\chi)(1+(\varphi-1)(1-\chi))}E_t\Delta g_{t+1}^n \\ &-\frac{\chi}{(1-\chi)}E_t\Delta g_{t+1}^n + \frac{(2\alpha_m-1)}{(1+(\varphi-1)(1-\chi))}E_t\Delta m\$_{t+1}^n \end{aligned}$$

Collecting like terms

$$\begin{aligned} r_t^n &= \rho + \frac{\varphi+1}{(1+\varphi(1-\chi))}E_t\Delta a_{t+1} + \frac{\chi}{(1-\chi)}E_t\Delta g_{t+1}^n\left(\frac{1}{(1+(\varphi-1)(1-\chi))} - 1\right) \\ &+\frac{(2\alpha_m-1)}{(1+(\varphi-1)(1-\chi))}E_t\Delta m\$_{t+1}^n \end{aligned}$$

Simplifying the coefficients for $E_t\Delta g_{t+1}^n$ and subtrating $\rho$ both sides we obtain the final expression for the deviation of the natural rate from the equilibrium rate,$\widehat{r}_t^n$ as shown below:

$$\begin{aligned} \widehat{r}_t^n &= r_t^n - \rho = \frac{\varphi+1}{(1+\varphi(1-\chi))}E_t\Delta a_{t+1} + \frac{\chi}{(1-\chi)}\frac{(1-(1+\varphi(1-\chi)))}{(1+(\varphi-1)(1-\chi))}E_t\Delta g_{t+1}^n \\ &+\frac{(2\alpha_m-1)}{(1+(\varphi-1)(1-\chi))}E_t\Delta m\$_{t+1}^n \end{aligned} \tag{A51}$$

The deviation of natural interest rate from the equilibrium rate is a function of expected natural government spending, technological advances; US dollar inflows.

Let $\phi_1 = \frac{\varphi+1}{(1+\varphi(1-\chi))}$, $\phi_2 = \frac{\chi}{(1-\chi)}\frac{(1-(1+\varphi(1-\chi)))}{(1+(\varphi-1)(1-\chi))}$, and $\phi_3 = \frac{(2\alpha_m-1)}{(1+(\varphi-1)(1-\chi))}$.

$$\widetilde{i_t} = \phi_1 E_t \Delta a_{t+1} + \phi_2 E_t \Delta g^n_{t+1} + \phi_3 E_t \Delta m\$^n_{t+1} \tag{A52}$$

**Fiscal Authority and Monetary Policy**

*Fiscal Policy*

This paper investigates government spending effects in a debt constrained econonomy. We assume for simplicity that government allocate expenditures to domestic goods to produce a public good. We denote the index of consumption for government goods by $G_t$ which is represented by the following expression;

$$G_t \equiv \left( \int_0^1 G_t(i)^{\frac{\varepsilon-1}{\varepsilon}} df \right)^{\frac{\varepsilon-1}{\varepsilon}} \tag{A53}$$

where,

$$G_t(f) \equiv \left( \frac{P_{d,t}}{P_t} \right)^{-v} G_t \tag{A54}$$

We introduce a countercyclical fiscal rule,that is a function of output, inflation and debt given by the following representation:

$$\widehat{G}_t = -\phi_y \widehat{Y}_t - \phi_\pi \pi_t - \phi_d \widehat{B}_t \tag{A55}$$

where, $\phi_y$,$\phi_\pi$ and $\phi_d$ are stabilisation parameters.

Government finances its consumption from lumpsum taxes and issuances of short term bonds to finance the government deficit. The mathematical representation of government debt gap is given by:

$$\widehat{B}_t = (1+i^*_t)(1+\delta)\widehat{B}_{t-1} + \widehat{G}_t \tag{A56}$$

where, $\widehat{B}_t$, represents the difference between governenmnet bonds issuance gap.

*Monetary Policy*

The small open economy is fully dollarised, interest rates are an unkown exogenous policy shock i.e $i^*_t = \rho^{i^*_t} a_{t-1} + \nu^{i^*_t}$.

Money supply is exogenous process. The supply of US dollars from the rest of the world economy is given by $ms\$_t = \rho^{ms\$} ms\$_{t-1} + \varepsilon^{ms\$}$.

**The equilibrium**

The main Characteristics of the small open economy in log linear form are given by:

1. The IS curve:

   Domestic economy

$$\widehat{y}_t = E_t \widehat{y}_{t+1} - (1-\chi)(i_t^* + \delta \widehat{b}_t - E_t \pi_{t+1} - \widehat{r}_t^n) - \chi E_t \Delta \widehat{g}_{t+1}$$

   where, $\widehat{r}_t^n$ denotes the deviation of natural interest rate from the equilibrium rate.

2. The Philips curve:

$$\pi_{d,t} = \beta E_t \pi_{d,t+1} + \lambda \frac{(1 + (\varphi - 1)(1-\chi))}{(1-\chi)} \widehat{y}_t - \frac{\lambda \chi}{(1-\chi)} \widehat{g}_t - \lambda (2\alpha_m - 1) \widehat{m\$}_t$$

3. Fiscal rule :

$$\widehat{g}_t = -\phi_y \widehat{y}_t - \phi_\pi \pi_t - \phi_b \widehat{b}_t$$

   where, $\phi_y$, $\phi_\pi$ and $\phi_b$, are stabilisation parameters.

4. Government debt is given by:

$$\widehat{b}_t = i_t^* + (1+\delta)\widehat{b}_{t-1} + \widehat{g}_t$$

   where, $\delta \widehat{b}_{t-1}$, repreesnts the risk premium on government bonds.

5. Domestic consumer price index

$$\pi_t = (1-\alpha)\pi_{d,t} + \alpha p_{f,t}$$

   Where $\pi_t$ is domestic inflation,$\alpha$ is the country's degree of openness and $p_{f,t}$ is foreign inflation.

6. Foreign inflation $p_{f,t}$ is represented as follows:

$$p_{f,t} = \rho^{p_{f,t}} p_{f,t-1} + \varepsilon^{p_{f,t}}$$

   where $\varepsilon^{p_{f,t}}$ is shock to foreign inflation.

7. Representation for output is shown below

$$y_t = \widehat{n_t} + \alpha_m \widehat{m\$_t} + a_t$$

where $n$ and $a$ and represents labour and productivity, respectively.

8. Demand for for US dollar holdings is given by:

$$md\$_t = \pi_{t,d} + \widehat{y_t} - \eta(i_t^* + \delta\widehat{b_t})$$

9. The deviation of natural interest rate from the equilibrium rate for the small open economy is given by:

$$\widehat{r_t^n} = \phi_1 E_t \Delta a_{t+1} + \phi_2 E_t \Delta g_{t+1}^n + \phi_3 E_t \Delta m\$_{t+1}^n$$

where, $\phi_1 = \frac{\varphi+1}{(1+\varphi(1-\chi))}$, $\phi_2 = \frac{\chi}{(1-\chi)} \frac{(1-(1+\varphi(1-\chi)))}{(1+(\varphi-1)(1-\chi))}$ ,and $\phi_3 = \frac{(2\alpha_m - 1)}{(1+(\varphi-1)(1-\chi))}$.

10. Money supply from the rest of the world economy is represented as follows:

$$ms\$_t = \rho^{ms\$} ms\$_{t-1} + \varepsilon^{ms\$}$$

where, $\varepsilon^{ms}$ is a shock to the supply of US dollars from the rest of the world economy.

11. Rest of the world monetary policy (i.e policy rate for the dollarised economy)

$$i_t^* = \rho^{i_t^*} i_{t-1}^* + \varepsilon^{i_t^*}$$

where $\varepsilon^{\widetilde{i^*}}$ is a shock to foreign interest rate.

12. Technology shock process is given as shown below

$$a_t = \rho^a a_{t-1} + \varepsilon^a$$

where, $\varepsilon^a$ is a shock to domestic productivity.

13. Interest rate gap

$$\widehat{r_t} = i_t^* + \delta\widehat{b_t} - E_t \pi_{t+1} - \widehat{r_t^n}$$

14. Real interest rate

$$r_t = i_t^* + \delta\widehat{b_t} - E_t \pi_{t+1}$$