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Co-Movement between Africa and International Stock Markets: Time-Varying Conditional Correlations with Wavelet Analysis

Abstract

We analyze the level of integration (co-movement) between Africa and international stock markets regionally and globally from 2002 to 2014, using the three-dimensional analysis of continuous Morlet wavelet, with consideration to the recent global financial crisis (GFC). Specifically, we address the following questions: (i) How well integrated are Africa's financial markets, regionally and globally? (ii) What is the nature and extent of integration (co-movement), if any? (iii) Was the level of integration influenced by the GFC? (iv). which major markets dominated in the integration processes? And, (v) How stable have the linkages been over time? Answering this question offers the chance to assess whether significant persistent changes in market linkages have happened across the markets analyzed. Since market co-movements can also lead to contagion, we further analyse to ascertain how cross-market linkages increased pre, during, and post-GFC. Contagion, according to Forbes and Rigobob (2002) is defined as significant increases in cross-market correlations after a shock compared to tranquil periods. Our determination of contagion therefore follows the above definition.

1. Introduction

Among other factors, with an anticipated human population growth of about 1.458 billion by 2025 (World Bank, 2014), Africa is increasingly becoming a frontier for investment and world economic development.[1] This calls for African countries to work out stronger ties and collaborations with the global economy. In the past three decades, efforts at integrating Africa regionally and globally have been aggressively pursued, albeit with some challenges. For instance, Africa has managed to significantly attain progress in economic integration including progressive development of regional infrastructure and removal of some barriers to intra-regional trade (Mougani, 2014). Despite this, progress in economic convergence, as well as, monetary coordination and financial sector integration remains slothful (Mougani, 2014). At the same time, lessons from the Eurozone suggest that efforts at attaining economic integration can better be enhanced on the wheels of prior monetary coordination and sufficient levels of financial convergence, regionally and globally. In African Development Bank's (AfDB) 2014 Policy Paper on the continent's Regional Integration, Litse and Mupotola (2014) recommend that the Eurozone model of economic integration should incite African Regional Economic Communities (RECs) to adopt measured and thoughtful approaches towards integration by meeting some basic conditions including financial sector convergence.

[1] Bodenhorn and Cuberes (2010) establish positive correlation between financial development and city growth robust to controls for city geographical characteristics, percentage of population working in different sectors, and initial population of a city.

Kawai and Motonishi (2005) suggest that measures of financial sector convergence can be categorized into three: price based measures (e.g. interest rate parity and stock markets), quality-based measures, and investment-based regulatory measures. Financial markets convergence in the philosophy of this study can be defined (in the context of price-based measures) as the harmonization and deepening of financial links through market structures to ensure an integrated financial system.[2] It implies, but not limited to the time-varying co-movements between stock markets, but also the nexus with commodities and currency prices. Such convergence of Africa's nascent markets are needful for some significant reasons: (i) to foster higher economic development through increased markets liquidity and lower cost of capital, (ii) to enhance informational efficiency, (iii) to overcome diseconomies, (iv) to condense the potential for arbitrage profits, etc – see also Furstenburg and Jean, (1989), Senbet, 2009, Ntim (2012), Asongu (2012), Coudert *et al.*, (2013), etc. On the flipside, financial markets convergence is noted to aid shocks contagion with consequences for markets stability. Again, markets integration may decrease the importance of the quality of securities regulation (Asongu, 2011) and make country specific factors less relevant in asset pricing (Bekaert and Harvey, 1995). It is argued that fear of vulnerability to the above adverse effects has led to most governments' reluctance to pursue programmes aimed at enhancing markets integration (Coudert *et al.*, 2013).

Inspired by initiatives such as the Abuja Treaty establishing the African Economic Community (AEC) on 12 May, 1994, the New Economic Partnership for Africa's Development (NEPAD), the Southern African Development Community (SADC), the West African Monetary Zone (WAMZ), and the East African Community (EAC), significant strides have been made to formally integrate and harmonize African stock markets regionally and globally. On the wings of this are the improvements in the overall equity market performances across the continent, though at a modest pace and levels below other emerging economies. With about 27 properly functioning exchanges (ASEA, 2013), total market capitalization of Sub-Saharan African (SSA) equity markets increased from US$605,113 million in 2005 to US$732,438 million in 2012. Of this, South Africa alone constituted US$565,408 million and US$612,308 million in 2005 and 2012 respectively. Although, South Africa has the highest market capitalization to gross domestic product (GDP) ratio in the sub-region, it recorded reduced values from 2005 (219.3%) to 2012 (154.1%). In SSA, total number of listed companies on all exchanges moved marginally from 911 (2005) to 923 (2012) compared to other emerging economies such as East Asia Pacific with 3,931 (5,311) and South Asia: 6,050 (6,496) in years 2005 (2012) respectively. In a similar fashion, by 2012, turn-over ratios (values of traded shares as a percentage of market capitalization) in SSA markets increased slightly from 37.3% in 2005 to 47.2% in 2012, anemic to that of East Asia Pacific of 68.4% (2005) and 127.7% (2012).[3]

Despite the significant rapid growth in their number and size, evidence abound to suggest that African stock markets remain generally segmented (Kodongo and Kalu, 2011; Ntim, 2012; Chinzara and Kambadza, 2014), small, illiquid and technologically weak (Ntim, 2012). At the same time empirical studies on the integration of the continent's stock markets regionally and

[2] The markets price harmonization process should however not be seen as the case of economic convergence criteria with a set of predetermined objectives.
[3] Unless stated otherwise, figures are gleaned largely from World Development Indicators Database (2015) - http://wdi.worldbank.org/table/5.4.

globally are scanty with mixed results. Although Alagidede (2010), Moss and Thuotte (2013), Ahmed and Mmolainyane (2014), Motelle and Biekpe (2015), and others have investigated Africa's financial integration to some extent and in diverse perspectives, considerable gaps exist to warrant serious research attention. First, it is not clear at the moment what factors have been driving the rather sparse integration of the African stock markets. Second, the role played by the 2007 global financial crisis (GFC) in moderating the regional and global integration of equity markets in Africa has not been profoundly investigated. Meanwhile, such development is likely to affect the level of cross-border listings of stocks and liquidity in the financial system with consequential effects on co-movements. On the basis of this, the present paper examines the nature and extent of African stock markets convergence, regionally and globally around the GFC. Particularly, the analysis is expected to point out whether there is a dominant regional or global market/economy influencing all other markets in Africa, and the significant factors at play. Additionally, the stability of the market nexus is investigated to account for the presence of any significant and persistent variations in the intensity of market co-movements across countries. Such analyses have useful implications for both portfolio selection and allocation decisions of investors, as well as for policy makers in surmounting the conundrums of Africa's financial markets integration agenda and shaping policy responses towards integrated and independent financial markets.

The anticipated contributions to extant literature underscore the significance and justification for conducting this study. First, converse to earlier studies in Africa which analyze stock returns co-movement; we examine the convergence of equity markets volatilities (for detailed review, see Nikkinen *et al.*, 2006; and Graham & Nikkinen, 2011). The rationale is that volatility quantifies the risk of a stock market, and therefore, is relevant to portfolio managers when rebalancing their portfolios from one market to another (Graham and Nikkinen, 2011). This logic is more grounded following the advent of the 2007 GFC that heightened market uncertainties and price fluctuations. The results therefore provide risk managers and policy makers with deeper comprehension of equity markets dynamics across geographical regions, thus helping them in devising effective hedging strategies. This makes our results robust to existing ones on African markets convergence.

The second contribution is that, opposing to existing studies that use data periods capturing few phases of the 2007 GFC; we use current and suitable data sets to investigate the integration of African markets. For example, the study explores the integration of African stocks for a full sample period, pre, during, and post-crisis periods. This handles problems inherent in the dearth of extant literature which use data sets that capture crisis periods that long predate the GFC. The intuition is that, reaction of market participants differ in periods of high and low market volatilities affecting the overall informational flow, cross-market listings, markets microstructures, and the degree and nature of co-movements.

The novelty of this study is also shown in the methodology employed. Most importantly, the use of wavelet estimation techniques which has relatively not seen substantial application on the African markets constitutes a significant advancement in the empirical studies on African stock markets convergence. Earlier and recent studies worldwide, have predominantly used cross-market correlation analysis (e.g. Longin and Solnik, 1995), various ARCH and GARCH models (e.g. Carrieri, *et al.*, 2007), and standard Granger causality or cointegration analysis (e.g.

Voronkova, 2004; Alagidede, 2010) as the metrics for equity markets integration. However, Pukthuanthong and Roll (2009) have vehemently criticized most of these models in modeling integration.[4] Perhaps, pioneered by Bekaert and Harvey (1995), there is a growing affinity of recent studies to employ techniques that account for time variation in estimating co-movements (see also, Gelos and Sahay, 2000; Bekaert *et al.,* 2008). Among the class of models gaining grounds in this respect in contemporary literature are wavelet techniques (see for example, Graham and Nikkinen, 2011; Madaleno and Pinho, 2012; Chakrabarty *et al.*, 2015; Chang and Lee, 2015, etc.). The uniqueness of the wavelet analysis is its localization in frequency band (time and scale), and ability to breakdown any ex-post variables on different frequencies to examine the subtleties of joint movements across diverse time horizons without losses in information. Thus, through wavelets we are able to make a distinction between the short-term and long term investor, as well as their investments horizons.[5] As opposed to standard Granger causality and cointegration techniques, wavelets are also able to overcome the problems of non-stationarity of the series.

The remainder of this paper is organized as follows. Section 2 presents a brief overview of stock markets in Africa. Section 3 outlines data and research design. Sections 4 and 5 present the results and conclusion respectively.

2.0 Data

All data used in this study are of daily periodicity on close-to-close basis and cover the period 3rd January 2003 to 29th December, 2014.[6] Daily prices or indices are analyzed using volatilities (based on absolute returns computed as the log difference between daily prices or indices) depending on the test under consideration. The study uses stock prices of eight African markets: Ghana, Nigeria, South Africa, Botswana, Morocco, Tunisia, Egypt, and Kenya. Additionally, prices of Morgan Stanley Capital International (MSCI) world index, which is comprised of developed world markets (hereafter referred to as MSCI developed markets index: (MSCI-DW)), MSCI emerging markets (MSCI-EM) index, Bloomberg Commodities (BCOM) index, and bilateral exchange rates between individual African countries on one hand, and each of the euro and dollar, on the other hand, are included in the sample. All data are gleaned from Thompson DataStream except BCOM, which is sourced from Bloomberg. Unless otherwise stated elsewhere, all indices/prices are expressed in U.S dollars, excluding the bilateral exchange rates with the euro.

In addition to the analysis of country-by-country convergence, we also estimate regional convergence. To do this, the African equity market data is aggregated into four regions of Africa represented in our dataset: East Africa (Kenya), West Africa (Ghana and Nigeria), Southern Africa (South Africa and Botswana), and North Africa (Tunisia, Egypt, and Morocco). Regional stock price/indices (computed as market or value-weighted average prices) are therefore constructed from individual markets indices based on the specific geographic distribution as

[4] The cointegration and standard Granger causality methodologies for instance, are limited to the stationarity assumption.

[5] As Candelon *et al.,* (2008) reports, from the stand-point of portfolio diversification, the short-term investor is usually interested in the integration of equity returns at higher frequencies (short-term fluctuations) while the long-term investor relies on linkages at lower frequencies (long-term fluctuations).

[6] We use daily data because lower frequency data (monthly or weekly) may not reflect all information on market interactions present in high frequency series.

indicated in Table 1. Including a stock from a given market in the regional index or price may result in upward bias or idiosyncratic market shocks in the regional index. For this reason, the valued-weighted regional index used for the bivariate estimations with each individual African market *i*, excludes that market, ostensibly to focus on shocks that are external to each market. Formally, the regional market valued-weighted index/price (p_t) excluding each individual market *i*, is computed as:

$$p_t = \sum_{t=1}^{T-i} w_{t,j} DPI_{t,j}^{q} \qquad [1]$$

where, q denotes any other market in the region, except i; $DPI_{t,j}^{q}$ is the daily price/index of market q in region j; w_t is the weight (which denotes the market capitalization) of each q, and T = total number of markets in a region. w_t is expressed as a fraction of the total market capitalization of all markets in the region. Because market capitalizations are of longer periodicity than daily indices, we use recently available end of year market capitalizations. All market capitalizations data are sourced from World Development Indicators (WDI, 2015) and the websites of the African Stock Exchanges Association (ASEA) and individual country specific stock exchanges.

2.1 Applying Wavelets to dynamically estimate time-varying convergence

A wavelet simply refers to a function with zero mean that is localized in both frequency and time. Wavelet methods have evolved overtime (since the early 1980's) and seen several applications in signal and image processing, medicine, geophysics, and astronomy; with some recent uses in economic analysis as an improvement over spectral density and Fourier techniques. Although spectral density plots (as indicated in [2.2.1] are able to detect frequency components that exist in the time series, it is unable to give information on the time localization of the different frequency parts. The Fourier technique operates under the hypothesis that the series is homogenous through time (that is, all active frequency components have the same amplitudes at all points). This however, is noted to be inefficient because the frequency resolution is the same across all different frequencies (Wen and Zeng, 2005). Wavelet methodologies were however developed as an alternative to the spectral density and Fourier techniques to allow for the estimation of the spectral features of a series as a function of time, revealing how the different periodic components vary over time (Madaleno and Pinho, 2009). Another distinctive feature of the wavelet transforms from the Fourier techniques is that the former are built over finite time intervals while the cosines and sines of the latter range from plus or minus infinity.

In this study, we apply a more recent package (WaveletComp) developed by Roesch and Schmidbauer (2014) to estimate the time-varying convergence of African stocks regionally and globally. In the package, wavelet functions are executed for easy accessibility of a wide range of intermediate and finite results. The WaveletComp can be applied to continuous wavelet analysis of univariate and bivariate time series with the null hypothesis that there is no (joint) periodicity in the series, which is tested through probability values from simulation. It provides extended plotting functionality to determine – (i) which objects are included to a plot (such as the ridge of wavelet power, contour lines indicating significant periodicity, arrows indicating the lead/lag series), (ii) the kind and degree of smoothing desired in wavelet coherence plots, (iii) the colour palette to use and definition of the lay-out of the time axis, etc. In our analysis we use the continuous wavelets (especially wavelet coherence to measure the extent of local correlation between two series in the time-frequency domain, and wavelet coherence phase differences) to analyze the frequency structure of the univariate and bivariate series.

2.1.1 The continuous-time Morlet wavelet transforms

Basically, wavelet transforms are of two categories: the continuous wavelet transforms (CWT) and the discrete wavelet transforms (DWT). Whereas the CWT is useful for extracting features, the DWT is mainly used for noise reduction and data compression. The continuous wavelet transform of a time series $W_x(s,\tau)$ can be defined as a convolution of the series with respect to a specific wavelet $\varphi(.)$ as:

$$W_x(s,\tau) = \int_{-\infty}^{+\infty} x(t) \frac{1}{\sqrt{s}} \varphi^*\left(\frac{t-\tau}{s}\right) dt \qquad [8]$$

where the term $1/\sqrt{s}$ denotes a normalization factor ensuring unit variance of the wavelet; and * refers to the complex conjugate form. The mother wavelet $\varphi(.)$ serves as a prototype for generating other window functions. The localizing parameter s (being shifted by a time increment of dt) determines the precise position of the wavelet whiles the scale parameter τ denotes the level of dilation. The wavelet is dilated if $(|\tau|>1)$ or compressed if $(|\tau|<1)$. The scale value is a fractional power of 2. Following Torrence and Compo (1998), we use Fast Fourier Transform (FFT) algorithms to evaluate equation [8].

The wavelet transform can be divided into real part $(\Re\{W_x\})$ and imaginary part $(I\{W_x\})$, thus providing information on both local amplitude $|W_x|$ and instantaneous phase $\tan^{-1}\left(\frac{I\{W_x\}}{\Re\{W_x\}}\right)$ of any periodic process across time – a condition precedent for the examination of coherency between two time series.

The mother wavelet is compressed or dilated (the extent of which is dependent on the scale parameter τ) to correspond to cycles of different frequencies when frequency information is to be extracted from the time series under consideration.

Characteristic of the CWT is the ability to decompose and reconstruct the time series $x(t)$ from its wavelet transform:

$$x(t) = \frac{1}{C_\varphi} \int_0^{+\infty} \left[\int_{-\infty}^{+\infty} W_x(s,\tau) \varphi_{s,\tau}(t) ds \right] \frac{d\tau}{\tau^2}, \quad \tau > 0. \qquad [9]$$

Additionally, CWT has the power to preserve the energy of the series under examination (Vacha and Barunik, 2012).

$$\| x \|^2 = \frac{1}{C_\varphi} \int_0^{+\infty} \left[\int_{-\infty}^{+\infty} |W_x(s,\tau)|^2 \, ds \right] \frac{d\tau}{\tau^2} \quad [10]$$

In this paper, all continuous wavelet transforms are executed using the Morlet wavelets. The Morlet wavelet allows for good identification and isolation of periodic signals, by providing a balance between localization of time and frequency (Grinstead *et al.*, 2004), and also appears to provide a better trade-off between detecting oscillations and peaks or discontinuities (Setz and Zurich, 2011). The Morlet wavelet, a plane wave modulated by Gaussian can be expressed in the simplest form as:

$$\varphi(\eta) = \pi^{-\frac{1}{4}} e^{i\eta\psi} e^{-\frac{\eta^2}{2}}, \quad [11]$$

where, η is non-dimensional 'time' parameter. The "angular frequency" ψ (or rotation rate in radians per unit time) is set to 6 to generate the admissibility of the Morlet function. The period or inverse frequency measured in time units is equal to $2\pi/6$, since one revolution equals 2π (radians). $\varphi(\eta)$ is complex, nonorthogonal, and normalized to have unit energy.

2.1.2 Wavelet power spectrum, coherency and phase difference

For proper examination of the time-varying relationship between two time series, we introduce a bivariate concept called the wavelet coherence. A better definition of the wavelet coherence can be attained by considering the cross-wavelet transform and wavelet power spectrum and phase difference. The concept of cross-wavelet analysis provides appropriate tools for (i) comparing the frequency contents of two time series, (ii) deriving conclusions about the synchronicity of the series at specific periods and across certain ranges of time – see Roesch and Schmidbauer (2014). The cross-wavelet transform is able to decompose the Fourier co- and quadrature-spectra in the time frequency (or time-scale) domain. Defined by Torrence and Compo (1998), the cross-wavelet transform (XWT) of two time series x_t and y_t can be specified as: $W^{xy} = W^x W^{y*}$; where W^x and W^y are the wavelet transforms of x and y, respectively, and * denotes a complex conjugate. In line with Valeda *et al.*, [2012] WaveletComp implements the rectified version given as:

$$W^{xy}(s,\tau) = \frac{1}{\tau} . W^x(s,\tau) . W^{y*}(s,\tau) \quad [12]$$

The modulus of equation [12] can be construed as cross-wavelet power – assessing the similarity of the two series' wavelet power in the time-frequency (or time-scale) domain (Roesch and Schmidbauer, 2014). It also shows the areas in the time-frequency space where the time series depicts a high common power, i.e. it denotes the local covariance between the time series at each scale (Vacha and Barunik, 2012). The wavelet power is given as:

$$P^{xy}(s,\tau) = | W^{xy}(s,\tau) | \quad [13]$$

P denotes wavelet power.

WaveletComp provides an image plot of the cross-wavelet power spectrum in the time-period domain, optionally with the cone of influence and with contour lines to indicate significance of joint periodicity or, for checks of consistency, joint significance of periodicity.

The phase for wavelet depicts any lead/lag linkages between two time series, and can be defined as:

$$\theta_{xy} = \tan^{-1}\frac{I\{W_t^{xy}\}}{\Re\{W_t^{xy}\}}, \qquad \theta_{xy} \in [-\pi, \pi] \qquad [14]$$

An absolute value of θ_{xy} less (larger) than $\pi/2$ indicates that the two series move in phase (anti-phase, respectively) referring to the instantaneous time as time origin and at the frequency under consideration, while the sign of the phase shows which series is the leading one in the relationship. In the graphical plots, the phase vectors are shown by arrows. Arrows pointing to the right suggest that the series are in phase. To the right and up with the first series lagging. Arrows to the right and down means the first series is leading. Arrows pointing to the left mean that the two series are out of phase. To the left and up shows the first series is leading. To the left and down shows that the first series is lagging – see also Madaleno and Pinho (2010, pp. 13-14) and Barbosa and Blitzkow (2008, pp.28-29) for more details.

Similar to Fourier coherency which measures the cross-correlation between two time series as a function of frequency, wavelet coherency is also considered as the equivalence of correlation coefficient, though there are significant differences between them (see Madaleno and Pinho, 2010, pp. 12). Wavelet coherency requires smoothing of both the cross-wavelet spectrum and the normalizing individual power spectra.[7] Wavelet coherency can be defined as the ratio of the cross-spectrum to the product of the spectrum of each series (Tiwari *et al.*, 2014). In line with Torrence and Webster (1998), we define the wavelet coherence of two time series *x* and *y* as:

$$R_t^2(s) = \frac{|S(s^{-1}W_t^{xy}(s))|^2}{S(s^{-1}|W_t^x(s)|^2).S(s^{-1}|W_t^y(s)|^2)} \qquad [15]$$

where *S* is a smoothing operator. It can be notice that the definition in equation [15] mimics the traditional correlation coefficient, and it is useful to think of the wavelet coherence as a localized correlation coefficient in time frequency and space (Madaleno and Pinho, 2010; Tiwari *et al.*, 2014). Wavelet coherence near one shows a higher similarity between the time series, whilst coherence near zero depict no relationship. We may express the smoothing operator, S as a convolution in time and scale:

$$S(W) = S_{scale}(S_{time}(W_t(s))) \qquad [16]$$

where S_{time} and S_{scale}respectively, denote smoothing in time and along the scale axis. For the Morlet wavelet a suitable smoothing operator is specified as:

$$S_{time}(W)|_s = \left(W_t(s)^* c_1^{-t^2/2s^2}\right)| \qquad [17]$$

and,

[7] Without smoothing, coherency is identically 1 at all scales and times.

$$S_{scale}(W)|_t = \left(W_t(s)^* c_2 \prod(0, 6s)\right)|_t \quad [18]$$

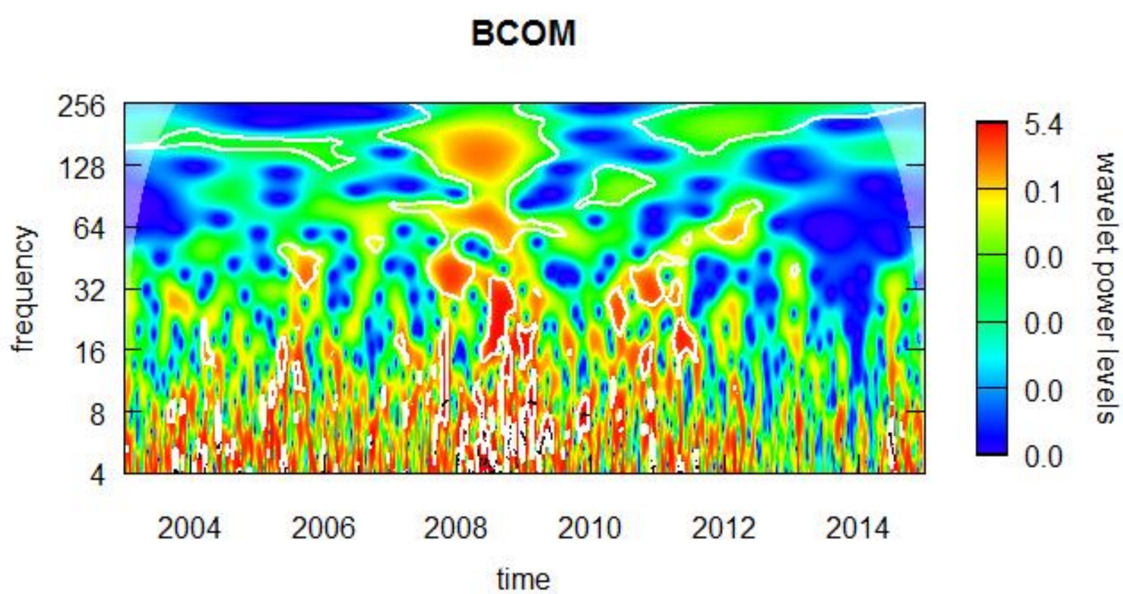
where c_1 and c_2 are normalizing constants and $\prod$ is the rectangular function. The factor of 0.6 is the empirically determined scale decorrelation length for the Morlet wavelet (Torrence and Compo, 1998).

3 Preliminary analyses

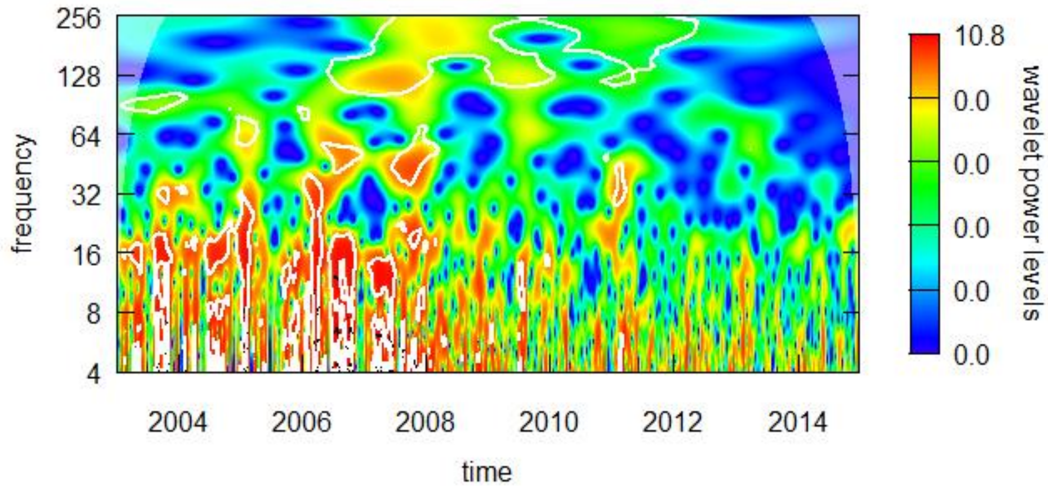
3.1 Empirical results of the wavelet power spectrum, coherency, and phase difference

Prior to the wavelets analysis, we present results of canonical correlations of all variables in Table 2 to examine the degree of association of African stocks with global and regional counterparts, as well as the bivariate exchange rates and the commodity index.

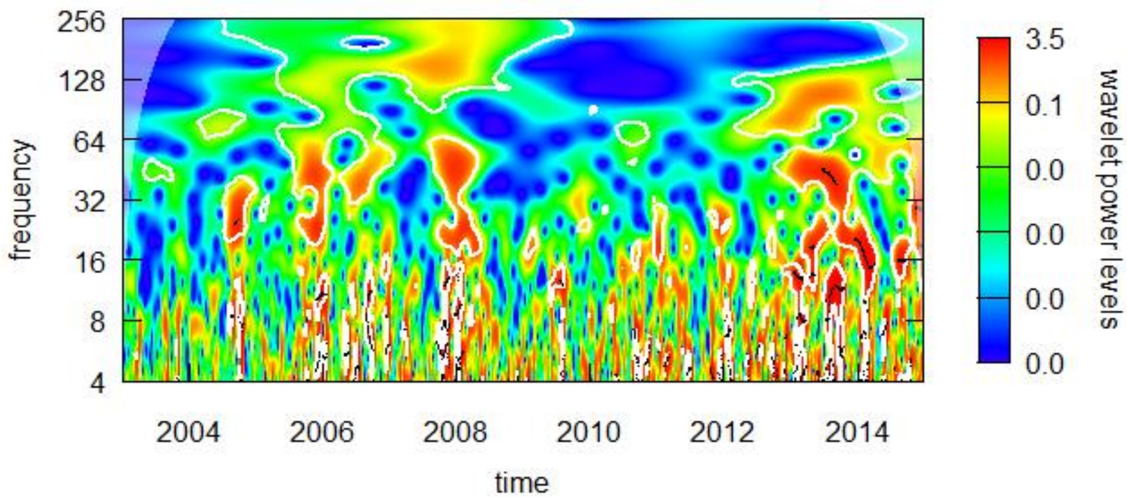
Our analysis of convergence with wavelets begins with results of the continuous wavelet power spectrum (WPS) of all considered individual and regional African markets, the MSCI-DW, MSCI-EM, BCOM, and the country-to-country bivariate exchange rates. The WPS is presented in plots with contours in time and frequency axes indicated on the horizontal and vertical axes respectively. Throughout this study, frequency is expressed in time units (years) for ease of interpretation. The frequency expressed in powers of two ranges from lower, 2 days (bottom of the plot) to upper, 256 days (top of the plot and approximate to one calendar year of trading). In the WPS, thick white contours in regions of energy denote significance at the 5% (95% confidence) level. Following a white noise process, the WPS is estimated from Monte Carlo simulations. To the right of the WPS is a colour bar depicting the steep power gradient of the significant contours ranging from blue (lower power) to red (higher power).
Periodicity

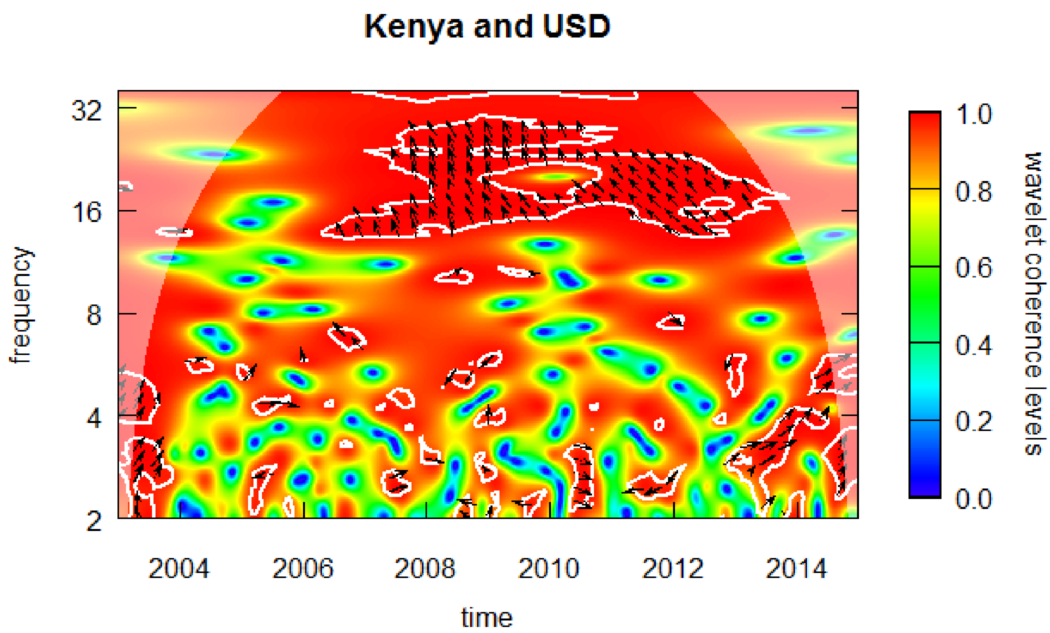


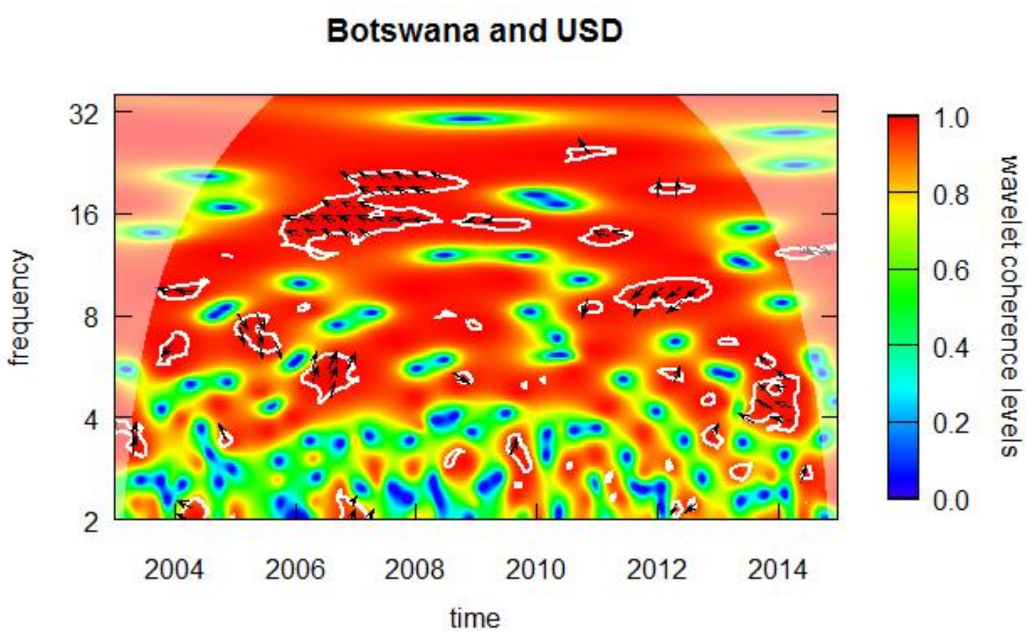
Botswana



Egypt




Kenya and USD
32
16
8
4
2
frequency
2004
2006
2008
2010
2012
2014
time
1.0
0.8
0.6
0.4
0.2
0.0
wavelet coherence levels


Botswana and USD
32
16
8
4
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frequency
2004
2006
2008
2010
2012
2014
time
1.0
0.8
0.6
0.4
0.2
0.0
wavelet coherence levels