AN EXTENDED EMPIRICAL ANALYSIS ON ASSET ALLOCATION IN THE BRICS STOCK MARKETS

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Abstract

Markowitz (1952) proposed mean-variance optimization for portfolio theory, whereby the goal is to maximize returns and minimize variance. This technique uses historical mean, variance and correlation. However, many studies have underlined the drawbacks of using these inputs. This paper uses three different optimization techniques, namely copula models that remedy the dependence structure between returns; multivariate GARCH models that remedy the static nature of the Markowitz optimization; and the Black-Litterman model which is forward looking instead of using a historical mean, and compares these models' performance to the benchmark, Markowitz model. Using the data from BRICS stock markets we find that all the models yielded higher risk-adjusted returns than the Markowitz model, however therefore proving empirical success in addressing the drawbacks previously highlighted in previous studies.

1. Introduction

Classic Economic theory suggests that individuals prefer more wealth to less, yet they are also willing to sacrifice some income in order to smooth out volatile income streams. This notion was well expressed in the work done by von Neumann and Morgenstern (1944) in their widely accepted descriptive theory of Expected Utility Theory (EUT) which postulates that decision makers have increasing concave utility functions for wealth under rational expectations. Indeed the issue of decision making under uncertainty has been a topic on the hot seat for policymakers, deal-makers, investors, firms and great economic scholars. This topic has seen great theories such game theory, risk aversion, and the prospect theory, to name a few.

Based on the then accepted descriptive theory of Expected Utility Theory, Modern Portfolio Theory (MPT) is concerned with portfolio selection and the process thereof may be divided into two stages. In his 1952 paper on Portfolio selection, Markowitz postulates that the first stage begins with observation and experience and concludes with beliefs about future expected returns on assets (Markowitz, 1952). He further asserts that the second stage deals with the optimal allocation of wealth. The notion of optimal allocation has since then taken the centre stage for research, both for academics and practitioners alike. Markowitz (1952) advocated that investors do not primarily desire to maximize their discounted expected returns, but that the primary objective is for the investor to minimize their risk given a certain level of returns. Vast literature has since been birthed in order to improve the performance of portfolios, after taking into account some of the pitfalls that were inherent to the Markowitz mean-variance portfolio optimization due to the underlying assumptions. Deng, Ma and Yang (2011) use the conditional value-at-risk (CVaR) as the measure of risk and apply the extreme value theory (EVT) to model the tails of the return series in order to improve the accuracy of the measurement of risk. They then apply pair Copula to describe the interdependence between assets, and thus construct the pair copula-GARCH-EVT-CVaR model for portfolio optimization. Further, Baptista and Gordon (2004) analyse the implications on portfolio optimization arising from imposing tighter constraints by means of the CVaR constraint. They postulate that the tighter constraint imposed by the CVaR is more effective as a tool to control slightly risk averse agents. Lauprete, Samarov and Welsch (2002) address the problem of the deviation from the normality assumption of asset returns. Moreover, they examine how the underlying estimation problem is influenced by heavy tails and multivariate tail dependence and model these by the univariate student-t distribution and the copula of a multivariate student-t distribution respectively.

In this paper, we compare the outlooks of portfolios composed with the same stocks, using different techniques that are suggested in the literature, and we evaluate the magnitude of their improvement on the mean-variance portfolio (if at all they improve on the mean-variance optimization).

The rest of the paper is organized as follows: sections 2 describes the methodology, section 3 reports our empirical analysis and findings and section 4 concludes the paper.

2. Methodology

2.1. Markowitz mean-variance optimization

The standard objective function to be maximized was pioneered by Markowitz (1952) when he postulated that investors desire not only to maximize their returns, but to further minimize their risk. This comes from the notion of mean-variance dominance, where portfolio A is said to mean-variance dominate portfolio B if, and only if, for the same level of risk, portfolio A has higher expected returns. Or alternatively, given the same level of expected returns, portfolio A has lower risk than portfolio B. Thus, efficient portfolios are all the portfolios which are not mean-variance dominated by any other portfolio. Now, let R_t denote the return of a portfolio at time t, R_t can be defined as

$$R_t = \ln(P_t/P_{t-1})$$

where P_t is the current asset price and ln is the natural logarithm. Consider a portfolio with n securities and the return on the ith asset is denoted by R_i. Let μ_i and σ_i^2 be the respective mean and variance corresponding to each ith asset, and let σ_{ij} be the covariance between R_i and R_j, and w_i is the weight of the wealth to be invested in asset i. It thus follows that the portfolio expected return and variance are as follows

$$\mu = E[R] = \sum_{i=1}^{n} \mu_i w_i = \boldsymbol{\mu}'\boldsymbol{w} \qquad (1)$$

where $\boldsymbol{\mu}' = (\mu_1, \mu_2, \dots, \mu_n)$ and $\boldsymbol{w} = (w_1, w_2, \dots, w_n)'$

$$\sigma^2 = Var[R] = \sum_{i=1}^{n}\sum_{j=1}^{n}\sigma_{ij}w_iw_j = \boldsymbol{w}'\boldsymbol{\Sigma}\boldsymbol{w} \quad (2)$$

where $\boldsymbol{\Sigma} = \begin{bmatrix} \sigma_1^2 & \sigma_{12} & \dots & \sigma_{1N} \\ \sigma_{21} & \sigma_2^2 & \cdots & \sigma_{2N} \\ \vdots & \vdots & \ddots & \vdots \\ \sigma_{N1} & \sigma_{N2} & \cdots & \sigma_{NN} \end{bmatrix}$

with

$$\sum_{i=1}^{n} w_i = \boldsymbol{1}'\boldsymbol{w} = 1 \quad (3)$$
$$w_i \geq 0, \quad i = 1, 2, \dots, n$$

where $\boldsymbol{1}' = (1, 1, \dots, 1)$.

The portfolio optimization problem then becomes

$$\min_{w} \frac{1}{2}\boldsymbol{w}'\boldsymbol{\Sigma}\boldsymbol{w} \quad (4)$$

Subject to

$$\boldsymbol{w}'\boldsymbol{\mu} = m$$

$$\boldsymbol{w}'\boldsymbol{1} = 1$$

$$w_i \geq 0$$

Using the Lagrange multipliers, the objective function to be optimized is thus as follows

$$F(\boldsymbol{w}, \lambda, \gamma) = \frac{1}{2}\boldsymbol{w}'\boldsymbol{\Sigma}\boldsymbol{w} + \lambda(m - \boldsymbol{w}'\boldsymbol{\mu}) + \gamma(1 - \boldsymbol{w}'\boldsymbol{1}).$$

Thus the optimal weight allocation is found by partially deriving F with respect to $\boldsymbol{w}$, λ, and γ and setting them equal to zero, such that

$$\frac{\partial F}{\partial \boldsymbol{w}} = \boldsymbol{\Sigma}\mathbf{w} - \lambda\boldsymbol{\mu} - \gamma\boldsymbol{1} = 0$$

$$\boldsymbol{\Sigma}\boldsymbol{w} = \lambda\boldsymbol{\mu} + \gamma\boldsymbol{1}$$

$$\boldsymbol{w} = \boldsymbol{\Sigma}^{-1}(\lambda\boldsymbol{\mu} + \gamma\boldsymbol{1})$$

$$\frac{\partial F}{\partial \lambda} = m - \boldsymbol{w}'\boldsymbol{\mu} = 0$$

$$m = \boldsymbol{w}'\boldsymbol{\mu}$$

$$m = \boldsymbol{\Sigma}^{-1}(\lambda\boldsymbol{\mu} + \gamma\mathbf{1})'\boldsymbol{\mu}$$

$$m = \lambda\boldsymbol{\mu}'\boldsymbol{\Sigma}\boldsymbol{\mu} + \gamma\mathbf{1}'\boldsymbol{\Sigma}\boldsymbol{\mu}$$

$$\frac{\partial F}{\partial \gamma} = 1 - \boldsymbol{w}'\mathbf{1} = 0$$

$$1 = \boldsymbol{w}'\mathbf{1}$$

$$1 = \boldsymbol{\Sigma}^{-1}(\lambda\boldsymbol{\mu} + \gamma\mathbf{1})'\mathbf{1}$$

$$1 = \lambda\boldsymbol{\mu}'\boldsymbol{\Sigma}^{-1}\mathbf{1} + \gamma\mathbf{1}'\boldsymbol{\Sigma}^{-1}\mathbf{1}$$

Now let $A = \boldsymbol{\mu}'\boldsymbol{\Sigma}\boldsymbol{\mu}$ $\quad B = \mathbf{1}'\boldsymbol{\Sigma}\boldsymbol{\mu}$ $\quad C = \mathbf{1}'\boldsymbol{\Sigma}^{-1}\mathbf{1}$

We then have it that

$$\lambda = \frac{Cm - B}{AC - B^2}$$

$$\gamma = \frac{A - Bm}{AC - B^2}$$

Hence the optimal weights are as follows:

$$w = \boldsymbol{\Sigma}^{-1}\left[\frac{(Cm - B)\boldsymbol{\mu} + (A - Bm)\mathbf{1}}{AC - B^2}\right] \qquad (5)$$

2.2. Mean-variance copula

The Markowitz mean-variance portfolio optimization postulates that investors benefit more from diversification when assets have low/negative correlation. However, it assumes that financial returns are multivariate-normally distributed and therefore that the dependence between financial returns is described by linear correlation coefficients. In contrast, empirical studies find evidence against the hypothesis of multivariate-normal distribution in financial returns and non-linearity in the dependence between financial returns (Boubaker and Sghaier). Using copula allows for the relaxation of the normality assumptions and further relaxes the assumption of linear correlation and can take into account the asymmetries in asymptotic tail dependence; whereby tail dependence refers to dependence that arises from extreme events between random variables.

The theory of copulas was introduced by Sklar (1959), and Sklar's theorem states that any joint distribution can be factored into cumulative density functions (CDFs) and a copula function that describes the dependence between components. Now let $x = (x_1, x_2)$ be a two-dimensional vector with joint distribution $F(x_1, x_2)$ and CDF $F_i(x_i)$, $i = 1, 2$. There exists a copula $C(u_1, u_2)$ such that

$$F(x_1, x_2) = P(X_1 < x_1, X_2 < x_2) = C\big(F_1(x_1), F_2(x_2)\big). \qquad (6)$$

Further, the theory states that if F_i are continuous, then $C(u_1, u_2)$ is unique. An Archimedean copula is one that admits the following representation:

$$C(u_1, u_2) = \begin{cases} \phi^{-1}(\phi(u_1) + \phi(u_2)) & if \sum_{i=1}^{2} \phi(u_i) \leq \phi(0) \\ 0 & otherwise \end{cases} \qquad (7)$$

The general relationship between Kendell's tau τ_C, which measures the concordance, and the Archimedean generator ϕ is expressed as

$$\tau_C = 1 + 4 \int_0^1 \frac{\phi(t)}{\phi'(t)} dt. \qquad (8)$$

In our optimization, we are interested in finding the copula parameter θ, which will replace the linear correlation coefficients ρ's in our variance covariance matrix. There are three types of Archimedean copulas which are in common use, namely the Clayton copula, Gumbel copula and the Frank copula, and we shall use the copula parameters acquired from these three techniques.

2.2.1. Clayton Copula

The Clayton copula is an asymmetric Archimedean copula that exhibits greater dependence in the negative tail than in the positive tail. This is a property which is common in financial data and this copula is given by

$$C_C(u_1, u_2) = \max\left[\left(u_1^{-\phi} + u_2^{-\phi} - 1\right)^{-\phi^{-1}}, 0\right] \qquad (9)$$

and it's generating function is

$$\phi(t) = \theta^{-1}\left(t^{-\theta} - 1\right) \ where\ \theta \in [-1, +\infty] \backslash \{0\} \tag{10}$$

The relationship between Kendell's tau and the copula parameter is given by

$$\hat{\theta} = \frac{2\tau_C}{1 - \tau_C}. \tag{11}$$

2.2.2. Gumbel Copula

The Gumbel copula is also an asymmetric Archimedean copula but exhibiting more dependence on the positive tail than the negative tail. This copula is given by

$$C_G(u_1, u_2) = e^{-\left[(-\ln(u_1))^{\theta} + (-\ln(u_2))^{\theta}\right]^{\theta^{-1}}} \tag{12}$$

and its generating function is given by

$$\phi(t) = \ (-\ln(t))^{\theta} \qquad where\ \theta \geq 1 \tag{13}$$

The relationship between Kendell's tau and the copula parameter is given by

$$\hat{\theta} = \frac{1}{1 - \tau_C}. \tag{14}$$

2.2.3. Frank Copula

The Frank Copula is a symmetric copula Archimedean copula and is given by

$$C_F(u_1, u_2) = \ -\frac{1}{\theta} \ln\left(1 + \frac{\left(e^{-\theta u_1} - 1\right)\left(e^{-\theta u_2} - 1\right)}{\left(e^{-\theta} - 1\right)}\right) \tag{15}$$

and its generating function is given by

$$\phi(t) = -\ln\left(\frac{e^{-\theta t} - 1}{e^{-\theta} - 1}\right), \quad where\ \theta \neq 1. \tag{16}$$

The relationship between Kendell's tau and the copula parameter is given by

$$\hat{\theta} = \frac{1}{1 - \tau_C}. \tag{17}$$

2.3. Mean-variance based on Multivariate GARCH

One way of addressing the nonlinear dependence between assets was to use the copulas and estimate the copula parameters that would replace the linear correlation coefficients in the variance covariance matrix, and the new covariance matrix would be optimized. In this section we introduce alternative methods based on multivariate generalized autoregressive conditional heteroskedasticity (MGARCH) models in order to address the issue of nonlinear interdependence.

We start by considering a stochastic vector process $\{\boldsymbol{r}_t\}$ with dimension $N \times 1$ and $E(\boldsymbol{r}_t) = 0$. We denote the information set related to $\{\boldsymbol{r}_t\}$ at time t as ξ_{t-1}. Assume that $\boldsymbol{r}_t$ is conditionally heteroskedastic, such that

$$\boldsymbol{r}_t = \boldsymbol{H}_t^{1/2}\boldsymbol{\eta}_t \qquad (18)$$

where $\boldsymbol{H}_t = [h_{ijt}]$ is the $N \times N$ matrix of the conditional covariance of $\boldsymbol{r}_t$, and $\boldsymbol{\eta}_t$ is and identically independently distributed (iid) vector error process, such that $E\boldsymbol{\eta}_t\boldsymbol{\eta}_t' = \boldsymbol{I}$. This is the standard framework for MGARCH, in which there is no linear dependence structure in $\{\boldsymbol{r}_t\}$. What remains to be specified is the matrix process $\boldsymbol{H}_t$.

2.3.1. DCC-GARCH

The Dynamic conditional correlation model takes into account the dynamic dependence between assets. In this model $\boldsymbol{H}_t$ is modeled using correlations coefficients that vary with time, and this resolves the problem of nonlinearity in the interdependence between assets. Thus for our optimization problem, our objective function will replace $\boldsymbol{\Sigma}$ with $\boldsymbol{H}_t$, where $\boldsymbol{H}_t$ is given by

$$\boldsymbol{H}_t = \boldsymbol{D}_t\boldsymbol{R}_t\boldsymbol{D}_t$$

$$\boldsymbol{D}_t = \begin{bmatrix} \sqrt{h}_{11,t} & 0 & \cdots & 0 \\ 0 & \sqrt{h}_{22,t} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \sqrt{h}_{NN,t} \end{bmatrix} \quad \boldsymbol{R}_t = \begin{bmatrix} 1 & \rho_{12,t} & \cdots & \rho_{1N,t} \\ \rho_{12,t} & 1 & \cdots & \rho_{2N,t} \\ \vdots & \vdots & \ddots & \vdots \\ \rho_{1N,t} & \rho_{2N,t} & \cdots & 1 \end{bmatrix}$$

2.3.2. GO-GARCH

The generalized orthogonal (GO) GARCH model uses $\boldsymbol{H}_t$ that may be obtained using the DCC-GARCH method and transforms $\boldsymbol{H}_t$ into an orthogonal matrix $\boldsymbol{V}_t$ through the use of a matrix of normalized eigenvectors $\boldsymbol{Z}_t$ of $\boldsymbol{H}_t$. By construction $\boldsymbol{Z}_t = [\boldsymbol{x}_1\ \boldsymbol{x}_2 \ldots \boldsymbol{x}_n]$, where $\boldsymbol{x}_i, (i = 1, 2, \ldots, n)$ denotes the normalized eigenvectors of $\boldsymbol{H}_t$.Therefore,

$$\boldsymbol{V}_t = \boldsymbol{Z}_t\boldsymbol{H}_t\boldsymbol{Z}_t'$$

We then use the orthogonal variance matrix $\boldsymbol{V}_t$ in our optimization problem in order to determine the optimal weights of our portfolio.

2.4. The Black-Litterman model

One of the basic assumptions of the Black-Litterman model is that an investor has specific views on securities. If not, then the investor may well just hold the equilibrium/market portfolio as it stands (Fabozzi et al. 2008). Let us define $\mathbf{\Pi}$ as our equilibrium returns, or alternatively, this is the vector of weighted expected returns from our standard mean-variance portfolio and $\boldsymbol{\mu}$ is the vector of the means of the returns of each asset in the portfolio.

$$\mathbf{\Pi} = \boldsymbol{\mu} + \boldsymbol{\varepsilon}_{\mathbf{\Pi}} \qquad (6)$$

$$\boldsymbol{\varepsilon}_{\mathbf{\Pi}} \sim N(0, \tau\mathbf{\Sigma})$$

where $\mathbf{\Pi} = (\Pi_{\mathbf{1}}, \dots, \Pi_{\boldsymbol{n}})'$ $\quad \boldsymbol{\mu} = (\mu_{\mathbf{1}}, \dots, \mu_{\boldsymbol{n}})'$ $\quad \boldsymbol{\varepsilon} = (\varepsilon_{\mathbf{1}}, \dots, \varepsilon_{\boldsymbol{n}})'$

$$\mathbf{\Sigma} = \begin{bmatrix} \sigma_{11} & \cdots & \sigma_{1N} \\ \vdots & \ddots & \vdots \\ \sigma_{N1} & \cdots & \sigma_{NN} \end{bmatrix}$$

We then incorporate the views of the investor, which may be absolute or relative. The absolute views may represent the views of the investor with regards to a single asset and its expected performance; however, the relative views are the investor's views about the performance of certain stocks in relation with the other stocks. Formerly, the K number of views are expressed in a K-dimensional vector $\boldsymbol{Q}$, with

$$\boldsymbol{Q} = P\boldsymbol{\mu} + \boldsymbol{\varepsilon}_{\boldsymbol{Q}} \qquad (7)$$

$$\boldsymbol{\varepsilon}_{\boldsymbol{Q}} \sim N(0, \mathbf{\Omega})$$

Where P is a $(k \times n)$ matrix that contains the assets with views, such that

$$P = \begin{bmatrix} p_{11} & \cdots & p_{1N} \\ \vdots & \ddots & \vdots \\ p_{k1} & \cdots & p_{kN} \end{bmatrix}$$

and

$$\mathbf{\Omega} = \begin{bmatrix} Var(\varepsilon_1) & 0 & \cdots & 0 \\ 0 & Var(\varepsilon_2) & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & Var(\varepsilon_k) \end{bmatrix}$$

$$Var(\varepsilon_k) = p_k' \mathbf{\Sigma} p_k$$

p_k is a single $(1 \times n)$ row vector of P corresponding to each k view.

Now we can "stack" equations (6) and (7) together as follows

$$\boldsymbol{y} = \boldsymbol{X\mu} + \boldsymbol{\varepsilon} \qquad (8)$$

$$\boldsymbol{\varepsilon} \sim N(0, \boldsymbol{V})$$

$$\boldsymbol{y} = \begin{bmatrix} \boldsymbol{\Pi} \\ \boldsymbol{Q} \end{bmatrix} \qquad \boldsymbol{X} = \begin{bmatrix} \boldsymbol{I} \\ P \end{bmatrix} \qquad \boldsymbol{V} = \begin{bmatrix} \tau\boldsymbol{\Sigma} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{\Omega} \end{bmatrix}$$

The generalized least squares solution for the parameter $\boldsymbol{\mu}$ yields the mean and the variance to be optimized for our portfolio selection. We thus have it that

$$\boldsymbol{\mu}_{BL} = \left(\boldsymbol{X}'\boldsymbol{V}^{-1}\boldsymbol{X}\right)^{-1}\boldsymbol{X}'\boldsymbol{V}^{-1}\boldsymbol{y}$$

$$\boldsymbol{\mu}_{BL} = \left(\begin{bmatrix} \boldsymbol{I} & P' \end{bmatrix} \begin{bmatrix} \tau\boldsymbol{\Sigma}^{-1} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{\Omega}^{-1} \end{bmatrix} \begin{bmatrix} \boldsymbol{I} \\ P \end{bmatrix}\right)^{-1} \begin{bmatrix} \boldsymbol{I} & P' \end{bmatrix} \begin{bmatrix} \tau\boldsymbol{\Sigma}^{-1} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{\Omega}^{-1} \end{bmatrix} \begin{bmatrix} \boldsymbol{\Pi} \\ \boldsymbol{Q} \end{bmatrix}$$

$$\boldsymbol{\mu}_{BL} = \left(\begin{bmatrix} \boldsymbol{I} & P' \end{bmatrix} \begin{bmatrix} \tau\boldsymbol{\Sigma}^{-1} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{\Omega}^{-1}P \end{bmatrix}\right)^{-1} \begin{bmatrix} \boldsymbol{I} & P' \end{bmatrix} \begin{bmatrix} \tau\boldsymbol{\Sigma}^{-1}\boldsymbol{\Pi} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{\Omega}^{-1}\boldsymbol{Q} \end{bmatrix}$$

$$\boldsymbol{\mu}_{BL} = \left[\tau\boldsymbol{\Sigma}^{-1} + P'\boldsymbol{\Omega}^{-1}P\right]^{-1}\left[\tau\boldsymbol{\Sigma}^{-1}\boldsymbol{\Pi} + P'\boldsymbol{\Omega}^{-1}\boldsymbol{Q}\right]$$

Since

$$\boldsymbol{Q} = P\boldsymbol{\mu} + \boldsymbol{\varepsilon}_{\boldsymbol{Q}}$$

we may thus express $\boldsymbol{\mu}_{BL}$ in terms of expected returns as

$$\boldsymbol{\mu}_{BL} = \left[\tau\boldsymbol{\Sigma}^{-1} + P'\boldsymbol{\Omega}^{-1}P\right]^{-1}\left[\tau\boldsymbol{\Sigma}^{-1}\boldsymbol{\Pi} + P'\boldsymbol{\Omega}^{-1}P\boldsymbol{\mu}\right]$$

and finally, the variance is given by

$$Var(\boldsymbol{\mu}_{BL}) = \left[\tau\boldsymbol{\Sigma}^{-1} + P'\boldsymbol{\Omega}^{-1}P\right]^{-1}.$$

Our optimization function remains same as the mean-variance optimization function given by equation (4), however we now replace the vector of means and the covariance matrix with the Black-Litterman mean vector and covariance matrix. The new optimal weights are given by equation (5) with the Black-Litterman mean vector and covariance matrix.

3. EMPIRICAL ANALYSIS

3.1. Data

For the construction of the portfolio, we used the returns on the composite equity indices of 5 emerging markets, namely Brazil, Russia, India, China and South Africa (BRICS), and one developed economy, United States of America. The objective was to have an internationally diversified portfolio among emerging market countries, while attempting to further diversify by including one developed economy.

Monthly data starting from December 1998 and ending in February 2013 was collected from I-Net Bridge. The returns series was generated using the log returns on the index prices.

For our Black-Litterman model, we were of the view that the South African returns would increase by 3.25%, with a 5% confidence. Further, we were of the view that China's stock markets would outgrow India and Brazil by 2.5%, with a 10% confidence. Our tau value was 0.5 to indicate our uncertainty level.

3.2. Empirical Results

Table 1: Optimal weights, expected returns, standard deviation and return-risk ratio of the constructed portfolio

	w1 (RSA)	w2 (BRL)	w3 (CHN)	w4 (IND)	w5 (RUS)	w6 (USA)	E[R] (%)	Stdev (%)	Ret/risk
Mean-variance	0.3008	0	0.132	0.0591	0	0.5082	0.5865	3.4251	0.1712
Clayton Copula	0.3345	0	0	0	0.3365	0.329	1.1169	5.481	0.2038
Gumbel Copula									
Frank Copula									
DCC LP	0.6579	0	0	0.1144	0.0254	0.2023	1.0124	4.3343	0.2336
DCC NLP	0.6154	0.0098	0.000	0.0672	0.0686	0.2391	1.0125	4.3398	0.2333
LP GO-GARCH	0.6579	0	0	0.1144	0.0254	0.2023	1.0124	4.3343	0.2336

NL GO-GARCH	0.6154	0.0098	0.000	0.0672	0.0686	0.2391	1.0125	4.3398	0.2333
Black-Litterman	0.3661	0	0.1288	0.0348	0	0.4703	0.631	2.1822	0.2892

The table of results includes all the techniques discussed in the methodology. However, in addition, we also used computed the linear and nonlinear convex programming techniques to solve our optimization problem in the DCC-GARCH and GO-GARCH models.

It can be observed from the table that the expected returns using the proposed extensions improved the expected returns of our Markowitz portfolio. However, the standard deviations of all the other techniques, in exception to the Black-Litterman Portfolio, were higher than the Markowitz portfolio standard deviation. Thus, as Markowitz (1952) postulated, rational investors are not only concerned with maximizing their expected returns, but they are also concerned about minimizing their risks. In line with that, we thus computed the return-risk ratio (=E[R]/Stdev), in order to see the ratio of the gain in comparison to the risk. Consequently, it is clearly evident that all the techniques do indeed improve on the Markowitz portfolio selection. Moreover, given our views, the Black-Litterman technique outperformed all methodologies.

4. CONCLUSION

The purpose of this paper was to evaluate whether the techniques proposed in the literature would indeed improve the optimal allocation of wealth in different assets. We confirmed that the proposed techniques were successful in improving the performance of the portfolio, but the improvement was by a small margin.

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